GEORGIA SCHOOL OF TECHNOLOGY
THE STATE ENGINEERING EXPERIMENT STATION
ATLANTA, GEORGIA

PROGRESS REPORTS NO. 1-5

PROJECT NO. 106-6

THE KEYING PROPERTIES OF QUARTZ CRYSTAL OSCILLATORS

CONTRACT NO. W36-039-sc-32100

VOL. I

BY

WILLIAM A. EDSON

JULY 12, 1946-JULY 31, 1947

GEORGIA SCHOOL OF TECHNOLOGY

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PROGRESS REPORT NO. I

PROJECT NO. 106-6

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CONTRACT NO. W36-039-sc-32100

BY

WILLIAM A. EDSON

JULY 12, 1946

SUMMARY

The full time services of two unusually able men have been secured for this project. In addition the services of two other qualified men are being sought.

The work is being performed in the electrical laboratory of the Physics Department of the Georgia School of Technology where considerable measuring and test equipment is available including many instruments which are seldom found in ordinary electrical laboratories. To supplement these facilities a number of special test instruments and a considerable number of electronic components are being procured.

The existing literature, both published and unpublished, is being searched for material pertinent to this project. When this material has been collected and studied, it will be combined into a tentative theory which will be checked by means of controlled experiments. New theories modified as required by these experiments, will be tested by further experiment and the theory-experiment process repeated until the controlling factors are evaluated.

PERSONNEL

At present the bulk of the work on this project is being done by Mr. Vernon R. Widerquist, a Graduate Engineer, and by Mr. Frederick Dixon, a Physics major. Neither man has had any considerable experience with quartz oscillators. Both are original thinkers of unusual calibre who will contribute fresh viewpoints to the investigation.

For the time being this staff is adequate to carry forward the program. However, a search is being made for an additional competent research worker and for a qualified technical assistant to supplement the group.

LABORATORY AND EQUIPMENT

The work on this project is being performed in the electrical laboratory of the Physics Department of the Georgia School of Technology. Therefore there is available for use on the project a considerable collection of various types of measuring instruments and equipment, including many instruments which are seldom found in ordinary electrical laboratories.

A special keyer unit capable of producing pulses of controlled duration and shape has been designed. It is believed that it will serve to represent the entire range of keying speeds met in practice. Details of this unit are shown in Figs 1 and 2.

The purpose of this unit is to provide an electronic means of keying the oscillator circuits under study. The keyer will have a continuously variable keying rate from one time per second to 10,000 times per second. In addition the keyer will provide a synchronizing voltage for an oscilloscope.

The time the oscillator is turned on or "gated" may be varied from a few microseconds per cycle to the maximum time per cycle permitted by the keying rate, that is, up to a maximum of one second. Also, the time position of the gate with reference to the synchronizing voltage may be varied. The rise and fall time of the gate voltage may be varied.

There are four coarse frequency ranges: 0.9 - 11, 9 - 110, 90 - 1100, 900 - 11000 cycles per second. A fine control over each of these ranges is provided.

An adequate stock of small parts such as resistors, condensers, coils, tubes, sockets, switches, etc. is being collected so that experimental oscillator and test units can be assembled as required without delay. No difficulty from this direction is anticipated.

OBJECTIVE AND GENERAL PROGRAM

The principal objective of the present study is the determination and evaluation of the factors which govern the performance of keyed crystal controlled oscillator circuits. To achieve this objective, the general program outlined below is being followed.

Satisfactory progress is being made in the preliminary phase, in which administrative procedures are established, personnel assigned, and the experimental facilities acquired. A system for keeping research records, which gives particular attention to completeness, clearness, and patent protection, has been adopted and personnel instructed in the use of this system.

The second phase involves the study of the pertinent literature, both published and unpublished. This information will be used to develop a tentative theory of the performance of keyed crystal oscillators. A thorough search of the literature to select all pertinent information is under way and will be followed by a detailed study of this information to develop a tentative theory of keyed crystal oscillator operation.

The third phase consists of experimental evaluation of the tentative theory. Undoubtedly there will be differences between theoretical and experimental results. The theory will then be modified and rechecked experimentally. This process will be continued until theory and experiment have been reconciled, after which an attempt will be made to reduce the results to provide a suitable design procedure.

OUTLINE OF WORK FOR SECOND QUARTER

The work of the second quarter will be directed primarily to complete the first and second objectives. Specifically the aims are:

1. To complete the search of published and unpublished literature and to extract the applicable material so that it will be readily usable for the project.
2. To review basic theory and to develop a special theory applicable to this problem.
3. To complete procurement of special laboratory and test equipment and electronic parts required by the project.
4. To design and construct special equipment required by the project.
5. To perfect laboratory procedures peculiar to this study.

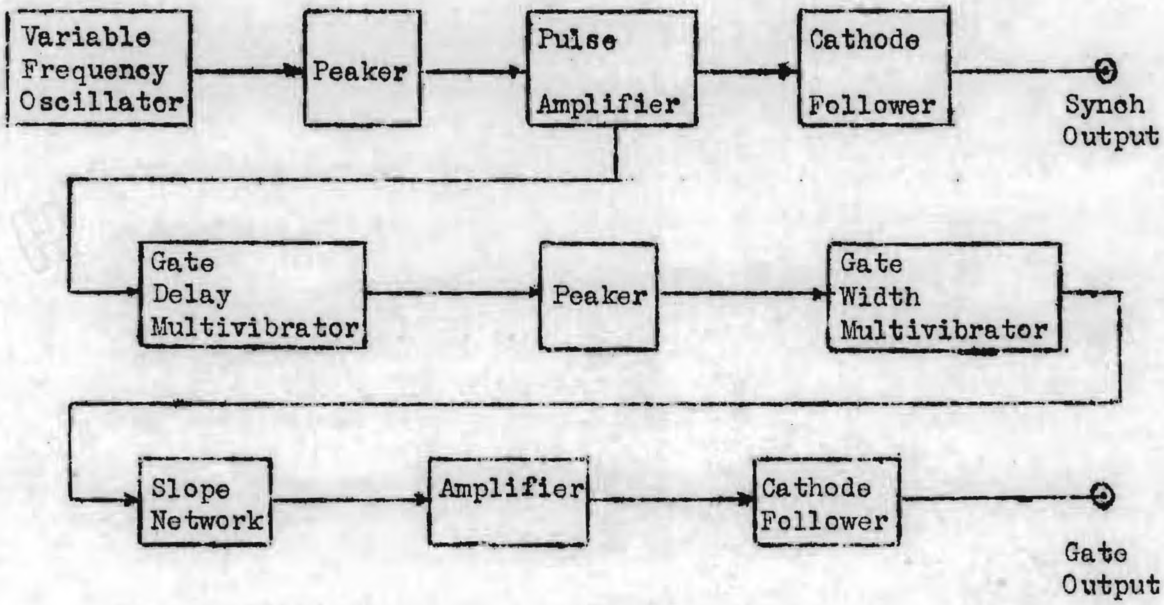
Respectfully submitted,

William A. Edson
Project Director

Gerald A. Rosselot
Director

Figure 1

BLOCK DIAGRAM OF KEYER UNIT

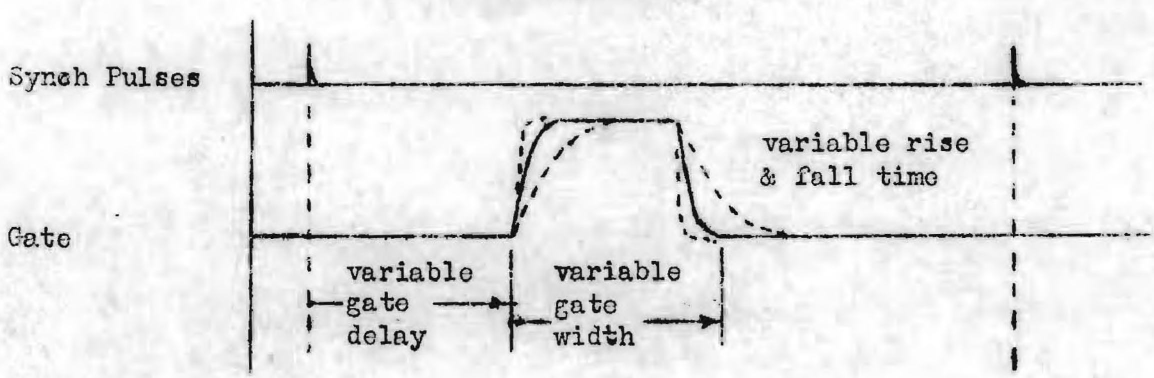


Keyer unit controls:

1. Coarse Frequency Range, 0.9-11, 9-110, 90-1100, &900-11000 cps.
2. Fine Frequency
3. Synch Amplitude
4. Gate Delay
5. Gate Width
6. Gate Amplitude
7. Leading Edge Slope
8. Trailing Edge Slope
9. ON-OFF

Figure 2

OUTPUT WAVE SHAPES



GEORGIA SCHOOL OF TECHNOLOGY

THE STATE ENGINEERING EXPERIMENT STATION

ATLANTA, GEORGIA

PROGRESS REPORT NO. 2

PROJECT NO. 106-6

THE KEYING PROPERTIES OF QUARTZ CRYSTAL OSCILLATORS

CONTRACT NO. W36-039-sc-32100

BY

WILLIAM A. EDSON

OCTOBER 12, 1946

SUMMARY

The services of a technical assistant have been secured to supplement those of the two men already engaged.

The project work is now being conducted in a new building erected especially for the joint occupancy of this and another electronic research project. The number of special instruments available to the project on a loan basis has been increased in the last three months. Moreover, a considerable stock of laboratory instruments has been accumulated for the exclusive use of this project.

The extensive search which has been made of the existing literature is being followed by a careful study of the relevant sections. The special theory applicable to the present problem is being further developed and will be tested by comparison with direct experimental results.

Special equipment for pulsing crystal oscillators has been contructed and tests will commence in the near future. A large oscilloscope has been especially modified to view the transients which occur at the beginning and end of the signal.

PERSONNEL

The same men mentioned in the last report, Mr. Widerquist and Mr. Dixon, continue to carry the bulk of the experimental load on this project. Their efforts are being supplemented by those of a technical assistant, Miss Catherine Yoe, a graduate of the Woman's College of the University of North Carolina.

LABORATORY AND EQUIPMENT

During this quarter the laboratory facilities of this project have been moved from the Physics Department to a special laboratory provided by the Engineering Experiment Station. This laboratory is in a building shared with another electronic research project and the building has been designed especially for electronic research. Adequate study and bench facilities are available.

Provision has been made for the continued use of special types of equipment available in the Physics Department and Electrical Engineering Department of Georgia Tech. In addition, the following specific instruments have been obtained for the exclusive use of the project:

1. General Electric Wide-band Oscilloscope.
2. Du Mont type 241 Oscilloscope.
3. Du Mont type 208 Oscilloscope.
4. General Radio Type 726A vacuum tube voltmeter.
5. General Radio Type 1800 vacuum tube voltmeter.
6. RCA type 195 vacuum tube voltmeter.
7. Du Mont type 185 Electronic Switch.
8. Keyer* designed for the purpose of keying oscillators.
9. Meter panels** to check oscillator performance.
10. Suitable voltage regulated power supplies.

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* Refer to Appendix I for explanation and circuit diagram.
** Refer to Appendix II for explanation and circuit diagram.

The stock of electronic parts has been considerably increased to the extent that all ordinary needs can be filled.

LITERATURE SEARCH

It is believed that the bulk of the literature pertinent to the project has now been catalogued. Some thirty books and about ninety articles from various periodicals have been abstracted. Most of these have been read with considerable care. A bibliography of the abstracted material is included in Appendix III.

About ten books have been chosen as most likely to contribute in an important way to the development of theory. These are to be studied in greater detail in the near future.

DEVELOPMENT OF THEORY

A review of the general problem of transients in linear systems has been made, since it appears that this material will be of primary importance to the project. Both the Heaviside and classical differential equation approaches have been used.

Transient theory has been applied to both simple and coupled circuits as a background for more advanced problems. The differential equations which govern the rise of oscillation in a grid-cathode crystal controlled circuit have been written and some progress has been made toward their solution.

A method for rapidly damping the ringing in a quartz crystal has been investigated. It is shown that by connecting a suitably dissipative coil across the terminals the damping can be increased by a factor of the order of 100.

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Project No. 106-6

OUTLINE OF WORK FOR THIRD QUARTER

The work of the third quarter will be a direct continuation of that which has just been completed. The specific aims are:

1. Further study of the material collected in the literature search.
2. Further development of a special theory applicable to this problem in terms of basic theory and information found in the literature.
3. Accumulation of experimental data on various types of crystals and circuits.
4. Tabulation and interpretation of this data in the light of theory.
5. Revision of laboratory equipment and techniques as dictated by experience.

Respectfully submitted,

William A. Edson
Project Director

Approved:

Gerald A. Rosselot
Director

APPENDIX I

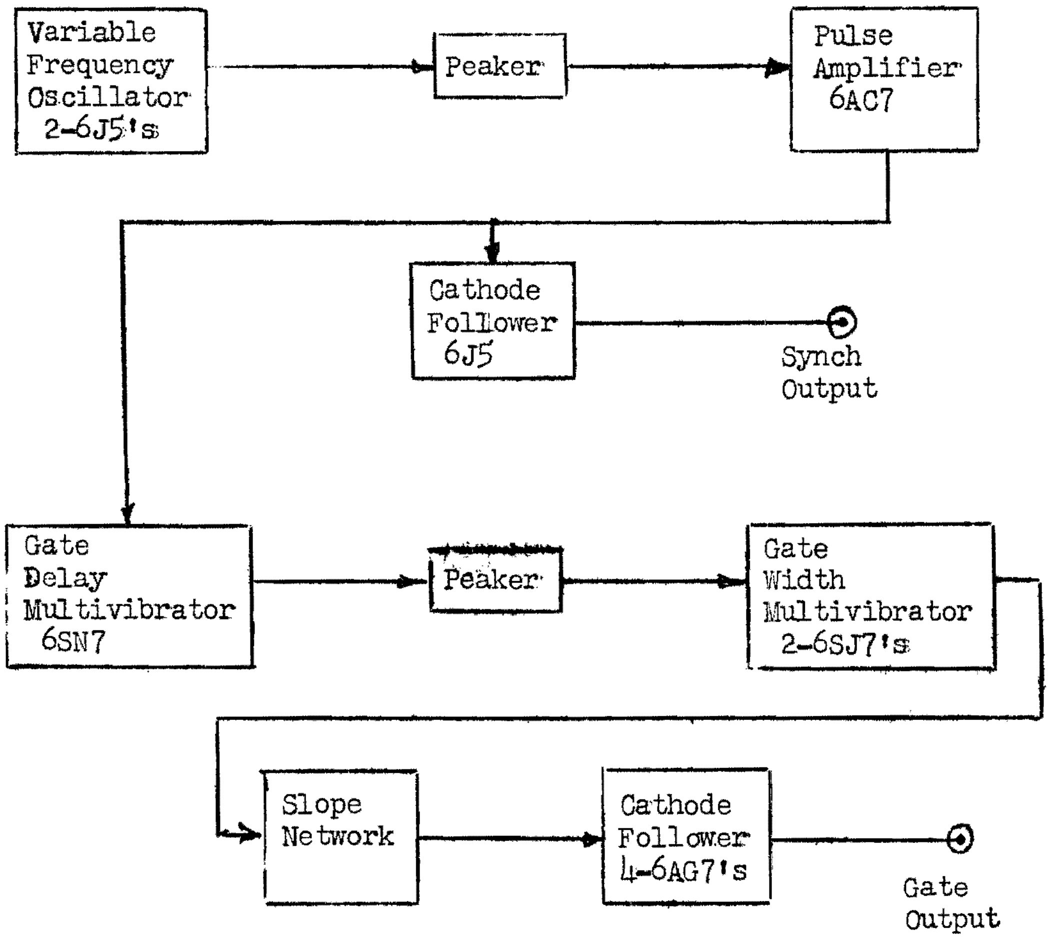
QUALITATIVE EXPLANATION OF THE OPERATION OF THE KEYER

The purpose of this keyer is to provide a keying pulse or gate with which to turn on and off the crystal controlled oscillator. Since the project requires a study of various keying rates, it is necessary for the keyer to have a variable repetition frequency. Furthermore, laboratory procedures render it advisable to make the keying gate variable in time with respect to a timing pulse. It is also desirable to make the keying gate width independently variable. Figure 1, attached to this appendix, is a block diagram of the keyer. Figures 2, 3, and 4 are the detailed schematic diagram of the keyer.

A qualitative explanation of the block diagram follows.

The variable frequency oscillator (Fig. 2) is a conventional positive grid return free-running multivibrator whose repetition rate is continually variable from 1 c.p.s. to 10,000 c.p.s. Coarse frequency control is provided by changing the value of the grid to plate coupling condensers. Fine frequency control is obtained by varying the grid return voltage. The square wave output of this multivibrator is peaked by a peaking circuit, and the resulting pulses are applied to a pulse amplifier which responds only to negative pulses. The output of this pulse amplifier is a positive pulse applied to a cathode follower which serves to provide a low output impedance and to isolate the pulse amplifier from external circuits. This positive output pulse, the purpose of which is to provide synchronization voltage for any external instrument, has values of approximately one microsecond duration and 100 volts amplitude.

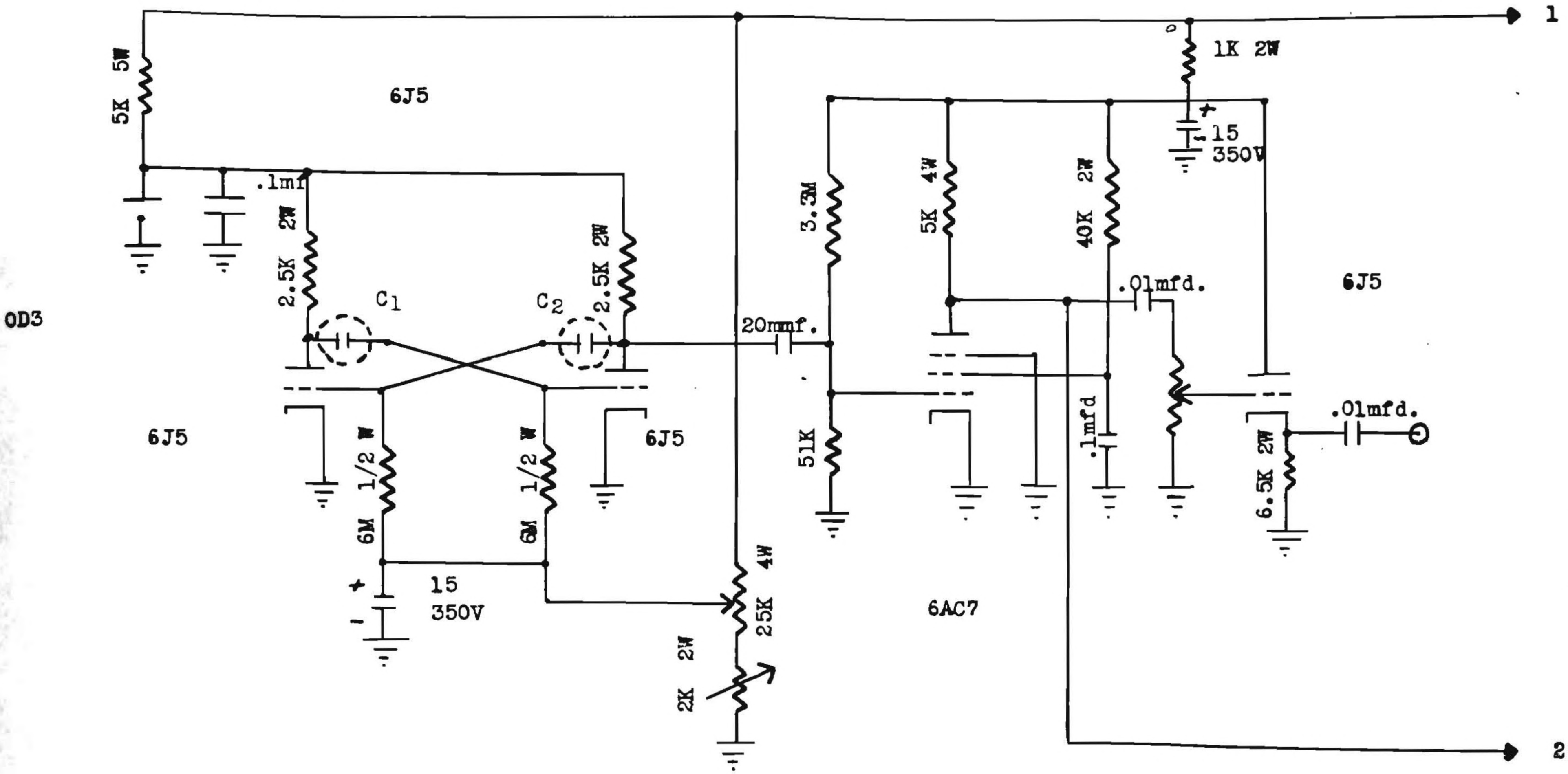
The output of the pulse amplifier is also used to trigger a gate delay multivibrator (Fig. 3) of the conventional start-stop type. The purpose of this multivibrator is to provide a variable time delay of the keying gate with respect to the synchronizing pulse. Thus, the keying gate may be set to occur when the synchronizing pulse occurs or may be set to occur a fixed time delay after the synchronizing pulse occurs. The output of the gate delay multivibrator is peaked and used to trigger a gate width multivibrator (Fig. 3) of the start-stop type. The gate output of this multivibrator is likewise continuously variable in width and provides the keying gate which is used to key the crystal oscillator. However, the output of the gate width multivibrator goes through a slope network (Fig. 4) which varies the rise and fall time of the keying gate. This modified gate is applied to a cathode follower (Fig. 4) which acts as an impedance matching device to an external circuit. Provision is made to change the output impedance of this section of the keyer so that the gate output may be used directly to furnish a peak output current up to 50 ma. without appreciably affecting the output voltage. A control is provided to vary the amplitude of the keying gate from 0 to 200 volts.



Controls:

Coarse Frequency
Fine Frequency
Gate Delay
Gate Width
Coarse Slope
Fine Slope
Synch Amplitude
Gate Amplitude
Output Impedance

FIGURE 1. BLOCK DIAGRAM OF KEYER

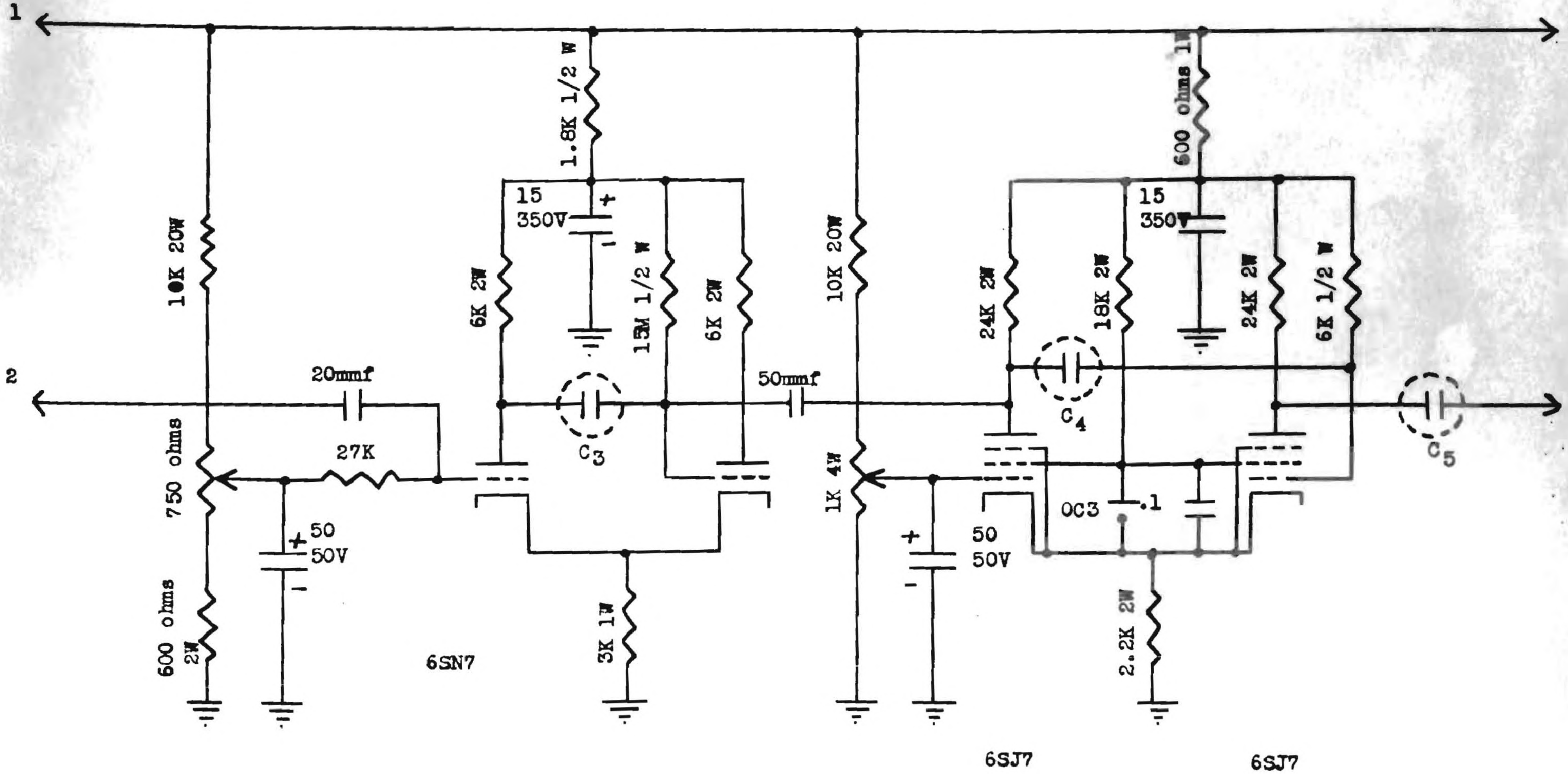


FREQUENCY RANGE	C_1	C_2
1-10	.1	.1
10-100	.01	.01
100-1K	.001	.001
1K-10K	.0001	.0001

All capacitances are in mfd. unless otherwise indicated

Fig. 2

	KEYER	
Drawn by	Checked by	Date
		9/26/46

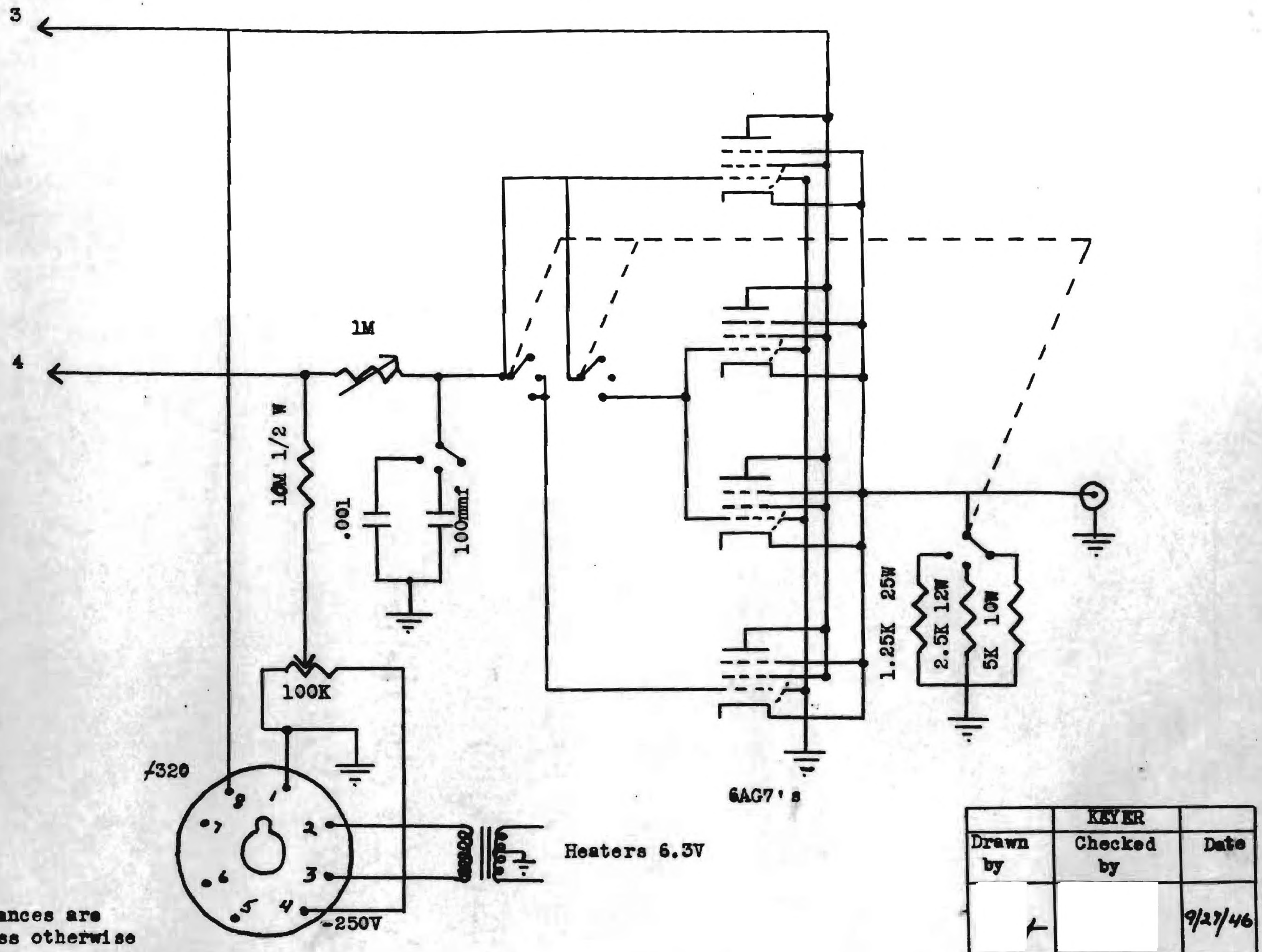


FREQUENCY RANGE	C_3	C_4	C_5
1-10	.5	.5	3.5
10-100	.05	.05	.5
100-1K	.005	.005	.5
1K-10K	.0005	.0005	.5

All capacitances are in mfd unless otherwise indicated.

	KEYER	
Drawn by	Checked by	Date
		9/27/46

Fig. 3



All capacitances are
in mfd unless otherwise
indicated.

Fig. 4

APPENDIX II

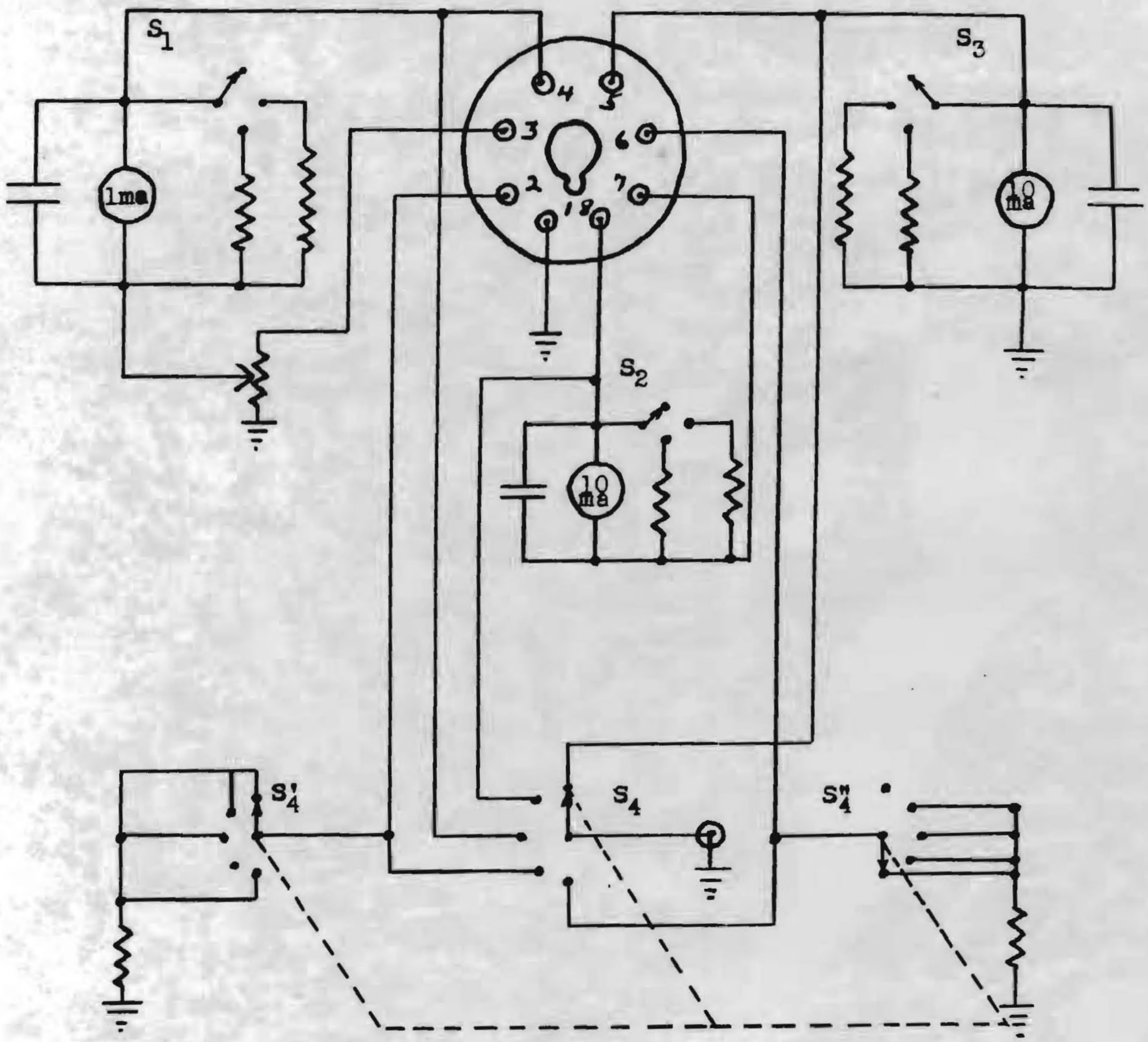
OSCILLATOR METER PANELS

When performing the experiments related to the study of keyed crystal oscillators, it will be necessary to make various voltage and current measurements on the oscillator circuits. To facilitate these measurements meter panels were designed for both triode, and pentode tubes.

When used with a triode, the panel permits direct and continuous measurement of cathode, grid, and plate currents. When used in conjunction with an external electronic voltmeter, measurements of grid bias, plate bias, and cathode bias, and the d. c. component of grid voltage and plate voltage, may be made by the use of a selector switch. A control is provided to vary the grid bias furnished to the oscillator.

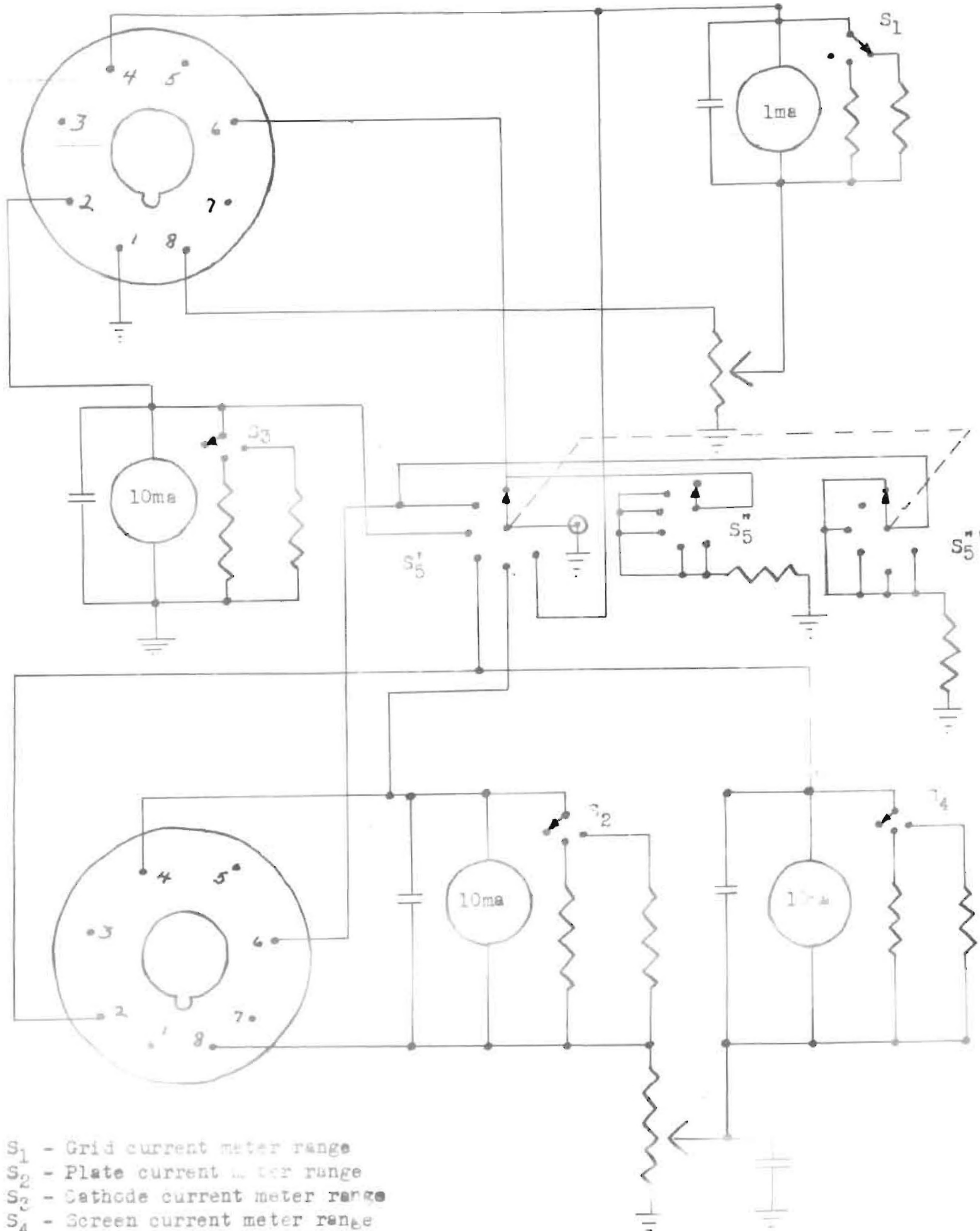
The pentode meter panel has, in addition to the above features, a screen current meter and a control for varying the screen bias furnished to the oscillator.

In both panels three current ranges are provided for each meter. The by-pass condensers for the meters are sufficiently large to by-pass any alternating currents present,



S_1 - Grid Current Meter Range
S_2 - Plate Current Meter Range
S_3 - Cathode Current Meter Range
S_4 - Voltage Selector Switch

TRIODE OSCILLATOR METER PANEL		
Drawn by	Checked by	Date
		9/27/46



S_1 - Grid current meter range
S_2 - Plate current meter range
S_3 - Cathode current meter range
S_4 - Screen current meter range
S_5 - Voltage selector switch

TETRODE OR PENTODE OSCILLATOR [illegible] PANEL

Drawn by	Checked by	Date
(		10/4/46

APPENDIX III

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GEORGIA SCHOOL OF TECHNOLOGY
THE STATE ENGINEERING EXPERIMENT STATION
ATLANTA, GEORGIA

PROGRESS REPORT NO. 3

PROJECT NO. 106-6

CONTRACT NO. W36-039-sc-32100

THE KEYING PROPERTIES OF QUARTZ CRYSTAL OSCILLATORS

By

WILLIAM A. EDSON

DECEMBER 28, 1946

SUMMARY

The personnel engaged in this project is the same as during the previous quarter. Similarly, the location and facilities are unchanged.

Detailed examination of the literature has disclosed a small amount of previous work directed toward the keying of quartz oscillators. However, nothing has been found which indicates a solution to the problem.

Preliminary experimental work indicates that very rapid keying may be accomplished if the crystal is allowed to ring continuously. However, this arrangement is not entirely satisfactory because the first few characters of a message will be distorted or lost and because break-in operation is endangered by such operation.

At the present time, it appears that adequate rates of build-up of oscillators may be achieved by use of high transconductance tubes in well-designed circuits, and that adequate rates of extinction may be achieved by application of damping resistance to the crystal. This approach will be followed during the next quarter.

Details of the work performed prior to December 10 were presented directly in conference at the Long Branch Signal Laboratory on December 10 and 11. A report summarizing these discussions is appended.

OUTLINE OF WORK FOR FOURTH QUARTER

The work during the fourth quarter will be a continuation of that which has just been completed. However, the emphasis will be somewhat more on direct experimentation than was formerly the case. Specific aims are:

1. Extension and re-examination of work done on determining the maximum rate of damping which can be secured by applying dissipative

networks across the terminals of a crystal. Theoretical results will be checked by direct experiment.

2. Compilation and study of literature pertinent to the optimum proportioning of cw crystal oscillator circuits as a basis for judging the performance of keyed oscillators.

3. Further search of literature directly bearing on the keying of crystal oscillators.

4. Extension of the theory of transient behavior of crystal oscillators by means of numerical examples, special cases, or other methods. One objective will be to determine the fastest build-up rate which can be achieved with existing tubes.

5. Theoretical and experimental study of the optimum waveform of keying as a basis for comparison of performance of actual circuits.

6. Direct experimental study of keying behavior of Miller and Pierce circuits.

7. Theoretical study of build-up rates as limited by available tubes, spurious crystal responses, frequency deviation, or other practical factors. Results will be checked by direct experiment.

Respectfully submitted,

[redacted]

William A. Edson
Project Director

Approved:

[redacted]
Director

APPENDIX I
PROGRESS REPORT NO. 3
CONTRACT NO. W36-039-sc-32100
PROJECT NO. 106-6

CONFERENCE REPORT NO. 1

A conference in connection with progress in the keying of crystal controlled oscillators was held at the Long Branch Laboratory of the Squier Frequency Control Laboratory on December 10 and 11, 1946. It was attended by the following personnel:

Long Branch Laboratory:

Messrs. E. W. Johnson
A. C. Pritchard
George Bower
C. B. Davis

Georgia Tech:

Messrs. W. A. Edson
V. R. Widerquist

The conference had as its two principal objectives the presentation by Georgia Tech of the work which has already been accomplished and an agreement by both groups as to the direction to be followed in future work.

RESULTS PRESENTED

Crystal Damping

A method for rapid damping of the ringing in quartz crystals was presented. A dissipative electrical network is connected to the terminals of the crystal so as to lower the net Q of the system during the time that oscillations are not desired. In a keyed oscillator this network would be connected in the key-up and disconnected in the key-down position.

In its simplest form this network consists of a single pure resistance. It is clear from consideration of the equivalent circuit of a crystal that the damping will be negligible if the resistance is either very low or very high. A simple analysis shows that maximum damping occurs when the resistance is equal to the reactance of the total shunting capacitance of the crystal and associated circuit. The resulting Q will be equal to or slightly less than twice the ratio of the two capacitances in the equivalent circuit.

Because the capacitance ratio for a quartz crystal cannot be less than 125, the Q can never be reduced appreciably below 250 by this method. At a frequency of one megacycle a Q of 250 results in a decay to .367 of initial amplitude in 95 microseconds. This would lead to satisfactory keying at about 100 dots per second.

Considerably higher damping rates may be achieved, at least theoretically, by use of a coil-resistance combination for the damping network. Present calculations indicate that optimum results are obtained with a high Q coil having an inductance of such value as to antiresonate the total shunting capacitance at the series resonant frequency of the crystal. Reduction of the effective Q to a value in the neighborhood of 10 is indicated.

Differential Equations.

A considerable amount of time has been devoted to a mathematical approach to the problem of keying. Both classical differential equation and Heaviside methods have been tried. In all cases so far studied, the differential equation which described the transient build-up of a crystal oscillator is of the fourth order with very complicated coefficients. Solution of the differential equation requires, in turn, the solution of an algebraic equation of the fourth degree.

Although it is true that methods exist for the formal solution of the general quartic equation, they are poorly adapted to any problem involving algebraic rather than numerical coefficients, and at best these methods are so tedious and bulky as to be impractical from the engineering viewpoint.

A large sheet summarizing the work that had been done in this connection was presented to the conference, but all present agreed that it did not constitute a promising approach to the problem.

Experimental Results.

An experimental Miller oscillator employing a 6J5 triode and operating at 200 KC was subjected to preliminary test. The circuit was keyed by a regularly repeated square wave in the B supply. Very acceptable waveforms were found in the plate circuit, even when the keying rate was as high as 7 KC. However, this does not constitute satisfactory performance. The crystal "rang" continuously at a nearly constant amplitude, and the tube served as a gate and an intermittent mechanism for providing the necessary feedback. Details of these experiments are shown in the form of oscilloscope tracings in Figures 1 and 2.

The principal objection to this sort of operation is that good performance is obtained after a sort of steady state is established. A considerable number of characters would be lost in bringing the oscillator, initially at rest, into this operating condition. This loss of information would be repeated each time the signalling was interrupted. An additional objection is that the radiation from the crystal and associated grid circuit might be sufficient to interfere with "break-in" operation.

It was agreed that an attempt should be made to operate in such a way that the crystal comes substantially to rest during each "key-up" interval.

DIRECTION OF FUTURE WORK

Keying Rates.

The original contract calls for a determination of maximum possible keying rates. This is a proper objective and continues as the ultimate goal.

However, the representatives of Georgia Tech found themselves without any practical reference with which to compare their theoretical and experimental results.

Keying speed is rated in terms of the standard word PARIS. Because this word with its accompanying space occupies the length of about 40 dots or 20 dot cycles, we may use the relationship 3 words per minute = 1 dot cycle per second in comparing message rates with recurrent square wave keying. Such a relationship is of importance because keying is most readily measured in terms of a recurrent square wave, but final performance is judged in terms of readable words per minute.

It developed as the conference proceeded that a variety of keying needs exist, but that two are of principal importance. Machine sending, which is used in connection with semi-permanent locations, is very fast, with a present maximum rate of one thousand (1000) words per minute, or 333 dot-cycles per second. Hand sending ranges from 10 to about 40 words per minute or from 3 to 13 dot-cycles per second. Standard teletype operates at 43 dot-cycles per second, which is substantially faster than the best hand or "bug" keying. The final decision was to make teletype speed (43 dot-cycles per second) the practical objective of this investigation. (Mr. Davis)

Keying Waveforms

It appears that for any particular keying speed there is an optimum pulse waveform. If the radio-frequency envelope rises and falls unduly rapidly, the signal is unpleasant to the listener, and adjacent channels suffer from excessive interference. These troubles are lumped under the heading of "key clicks." If, on the other hand, the rates of rise and fall are too low, the signals become blurred. Dots do not rise to full amplitude, and spaces do not correspond to complete silence.

No quantitative data were available, but the conference representatives agreed that the rise and fall times of the pulse should represent an appreciable portion of the dot duration. Work to obtain a practical engineering answer to this question is being planned.

Oscillator Circuits.

The advantages of the Pierce, Miller, and other crystal circuits were discussed. Many factors are involved, and the situation is complicated, but a few facts are outstanding. The Pierce circuit is much favored for apparatus in which frequency changes must often be made because good operation may be secured over a frequency ratio of as much as 15:1 without retuning.

Representatives of Georgia Tech advanced the opinion that the Miller circuit leads to larger values of power output at a given crystal dissipation than the Pierce. However, there appear to be data to the contrary, and no agreement on the point was reached.

Because of their simplicity, the Pierce and Miller circuits will receive primary attention. However, it is acknowledged that these simple circuits may not key properly. In such event, an examination will be made of more complicated circuits such as the Meacham, C. I. meter, or cathode-coupled oscillators. It was concluded that if these complicated circuits key properly they would be suitable in spite of the increased difficulty of tuning. (Mr. Pritchard)

Spurious Responses.

The use of high transconductance tubes in properly proportioned circuits appears desirable in the interest of obtaining the rapid rates of build-up required for fast keying. Two drawbacks to this procedure are anticipated. First is a tendency toward excessive production of harmonics. The conference agreed that this is not likely to be a serious drawback. More serious is a tendency for the circuit to operate at frequencies corresponding to spurious responses of the crystal. It is quite possible that two or more frequencies will be produced during the rise interval even though only one will be produced in the steady state because of the transition from linear to non-linear operation of the tube.

Definite numbers were not given, but it was the consensus of opinion that the activity of spurious modes will not exceed 10% of that of the desired mode, and that 1% will be the limit in most cases. Moreover, the frequency of serious spurious modes is generally well removed from that of the desired mode.

Choice of Tubes.

The necessity of rapid build-up recommends the use of high gain tubes. General preference was expressed for pentodes and beam-tetrode tubes.

Crystal Dissipation.

In its program to standardize the design and application of crystals, the Squier Laboratory plans to set an upper limit on the power which may be dissipated in a crystal. The tentative figure for cw oscillators is 100 milliwatts. (Mr. Davis) It is planned to use the same figure on the basis of continuous operation in the design of keyed oscillators.

Frequencies.

The frequency range to be studied is given in the original contract and was reaffirmed in the conference. No developments have yet occurred to modify the range originally specified. It was recommended that no examination be made of crystals operating at overtones of their principal resonant frequency.

Gating.

It is relatively easy to construct a crystal oscillator which operates continuously at a very stable frequency. Signals are obtained by following such an oscillator with a keyed or gated amplifier. Suitable signals are readily obtained by this method even at the highest keying speeds now visualized. However, two serious drawbacks exist. An additional tube and tuned circuit are usually required, so that the size and complexity of the equipment are increased. More serious, the oscillator, which operates continuously, radiates enough energy to interfere with the accompanying receiver tuned to the same frequency. This precludes "net operation" with break-in. Careful design and shielding will probably solve this difficulty, but at the expense of still further weight, bulk, and complexity.

It was the opinion of the conference that gating will be necessary where machine sending is employed and that in this connection the extra bulk and complexity are not serious objections because of the relative complexity of the associated equipment. It seems desirable that the problems involved in gating be given further study. Such work is, however, outside the scope of the present project.

EQUIPMENT

A general inspection of the laboratories was made to exchange ideas and opinions in connection with apparatus and measuring techniques. The tour was quite helpful to the representatives of Georgia Tech.

C. I. Meter.

Models of the crystal impedance meter were inspected for details of mechanical construction. The corresponding circuit diagrams were studied for design detail and philosophy, and the technique of operation was discussed. The procedure for determining the equivalent circuit of a given crystal is as follows:

1. By use of the C. I. meter, measure directly the resistance of the crystal at series resonance.

2. By use of a Q meter or bridge, measure directly the shunting capacitance.

3. By means of a suitable stable oscillator, measure the resonant and antiresonant frequencies as judged by the fact that the crystal is a pure resistance at both frequencies.

Two of the four elements of the crystal equivalent circuit are determined directly by the first two measurements. The series capacitance is next determined by the fact that the ratio of the two capacitances in the equivalent circuit depends only on the ratio of the antiresonant and resonant frequencies. Finally, the inductance is calculated from the known series capacitance and the series-resonant frequency.

G. F. E.

The method of securing Government Furnished Equipment for this project was discussed. A satisfactory understanding of procedures was reached, and letters in this connection have already been written.

William A. Edson
Project Director

Georgia School of Technology
STATE ENGINEERING EXPERIMENT STATION
Atlanta, Georgia

PROGRESS REPORT NO. 4

PROJECT NO. 106-6

CONTRACT NO. W36-039-sc-32100

THE KEYING PROPERTIES OF QUARTZ CRYSTAL OSCILLATORS

By

WILLIAM A. EDSON

APRIL 12, 1947

GEORGIA SCHOOL OF TECHNOLOGY

THE STATE ENGINEERING EXPERIMENT STATION

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Progress Report No. 4
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TABLE OF CONTENTS

SUMMARY

During this quarter, a large amount of work was devoted to the theory of damping the vibrations of a quartz plate. The results, which are presented at length in the following sections, lead to the conclusion that damping is quite practical and is necessary for high keying rates. Damping which is rapid enough for most applications may be secured by use of a single resistor, which will operate satisfactorily over a considerable band of frequencies. However, a frequency shift which may be objectionable occurs when the resistor is applied. Greater damping rates may be obtained by the use of resistance in combination with inductance. It is possible to proportion these elements so that no frequency change occurs during the decay interval. The analysis has not yet determined how wide a band of frequencies can be covered by substitution of crystals without requiring readjustment of the damping network. However, there is reason to believe that the limits are not narrow.

In order to secure rapid build-up rates it is necessary to employ tubes having large values of transconductance in circuits favorable to oscillation. It is found that available tubes have adequate values of g_m to reach signalling speeds in excess of 50 dot-cycles per second if the circuit values are properly chosen. When electron coupling is used, it is necessary to use considerable care in the choice of the circuit, or a large loss of effective transconductance will occur.

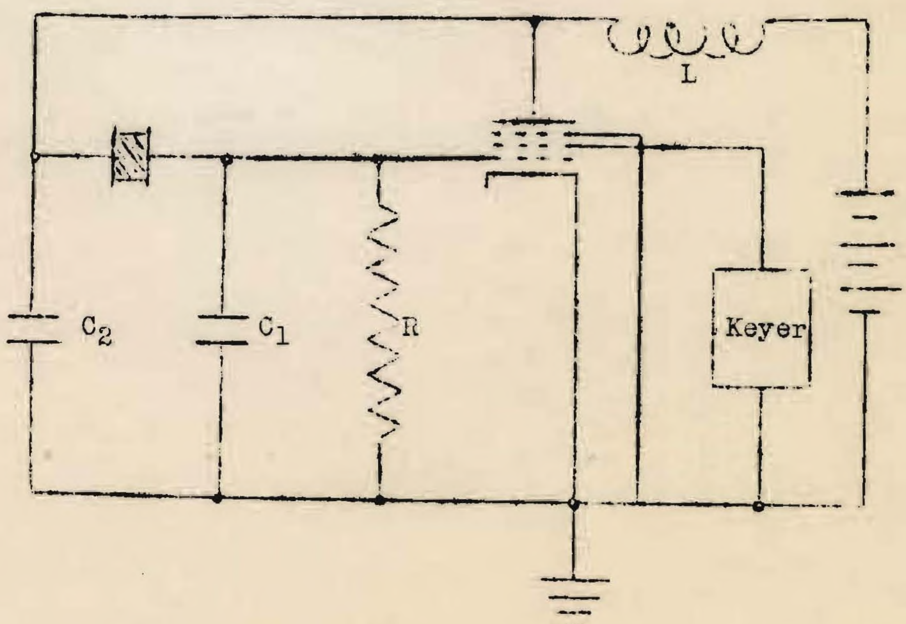
The effect of waveform on the intelligibility has been studied experimentally as a guide to the design of keying circuits. It is tentatively recommended that the signal shall rise to 80% of its maximum value in less than 25% of the period of one dot-cycle and should decrease to 20% of maximum value in less than 10% of the period of one dot-cycle.

A circuit for operation at the series resonant frequency of the crystal is proposed. The relative merits of several forms of electron coupled oscillators are also discussed.

I. PRELIMINARY EXPERIMENTS ON THE PIERCE OSCILLATOR

It is evident that an examination of the keying characteristics of crystal controlled oscillators will present many problems, some of which may be easily determined by a qualitative examination. Other problems, however, may appear only after a detailed theoretical or experimental study. In an attempt to determine and evaluate some of these "hidden" problems and to develop some familiarity with the peculiarities of crystal oscillators, a series of experiments was performed on a Pierce oscillator.

The basic Pierce oscillator circuit is shown in Figure 1. For experimental purposes, it was desirable to use the simplest possible form of the circuit. Certain values of the circuit elements were selected to present a proper load capacitance to the crystal; the others because they gave a good stable oscillator. The circuit in Figure 1 is designed to use a crystal requiring a load capacitance between 25 and 30 $\mu\mu F$.



$C_1 = 50 \mu\mu F$
$C_2 = 50 \mu\mu F$
$R = 3.9M$
$L = .75$ mhys.

Figure 1.
Simple Pierce Oscillator

The tube used was either a 6AC7 or a 6L7 depending upon the particular phase of study. Using a 6AC7 the oscillator was found to be stable over the available range of plate supply voltages (210 to 350 volts) and screen grid supply voltages (105 to 350 volts, the plate supply being maintained at a D.C. voltage greater than the screen supply). The amplitude of oscillation was found to be independent of the plate supply voltage but was affected by the screen supply voltage.

The discussion of keying characteristics is facilitated by the use of two definitions:

(a) The rise time (abbreviated R.T.) is defined as the time required for the plate oscillations to reach 63.6% of the maximum value.

(b) The fall time (abbreviated F.T.) is defined as the time required for the plate oscillations to drop to 36.4% of the maximum value.

To improve high speed keying it is necessary to determine the limitations imposed on R.T. and F.T. by various circuit parameters and then develop a method of overcoming these limitations.

Variation of crystal Q.

Because quartz crystals have a very high Q, the crystal will undoubtedly be one of the prime factors in producing a long R.T. and F.T. The initial experiment was performed to determine this effect. Five crystals (whose equivalent parameters were known) were used in the oscillator, and the R.T. and F.T. were measured for each crystal. The data obtained are tabulated in Table 1. From the data shown in Table 1, it can be seen that both R.T. and F.T. increase as Q increases; moreover, the fall time is almost proportional to Q.[1]

Crystal No.	Crystal Q.	Rise Time	Fall Time
15	104,000	3.0 ms.	10.0 ms.
12	66,200	2.6 ms.	7.1 ms.
14	56,300	2.5 ms.	7.6 ms.
11	50,400	1.9 ms.	5.4 ms.
13	48,600	1.9 ms.	5.0 ms.

Table 1.
Correlation of Rise Time and Fall Time with Q.

Variation of capacitance ratio.

Referring to Figure 1, it is seen that the ratio of C_1 to C_2 may be varied and not appreciably affect the capacitive load presented to the crystal. This statement will hold if $\frac{C_1C_2}{C_1 + C_2} = K$, where K is the load capacitance for that particular crystal. It can be shown for the Pierce oscillator that the g_m required for sustained oscillations is a minimum when $C_1 = C_2$; it follows that the rise time will be a minimum when $C_1 = C_2$.

It was desired to verify these statements experimentally and to find the limits within which the ratio C_1/C_2 may be varied without seriously affecting the rise time.

The oscillator was set up using a 6L7 with a g_m of approximately 1000. The data obtained are tabulated in Table 2 and plotted in Figure 2.

The time delay is arbitrarily measured in distance on a linear scope trace. These data are plotted in Figure 2 using C_1/C_2 as the abscissa and time delay as the ordinate. From this graph it can be seen that C_1/C_2 may be

(1) Subsequent experiments indicate that the Q of crystal No. 12 is probably about 50,000 rather than the calculated value of 66,200.

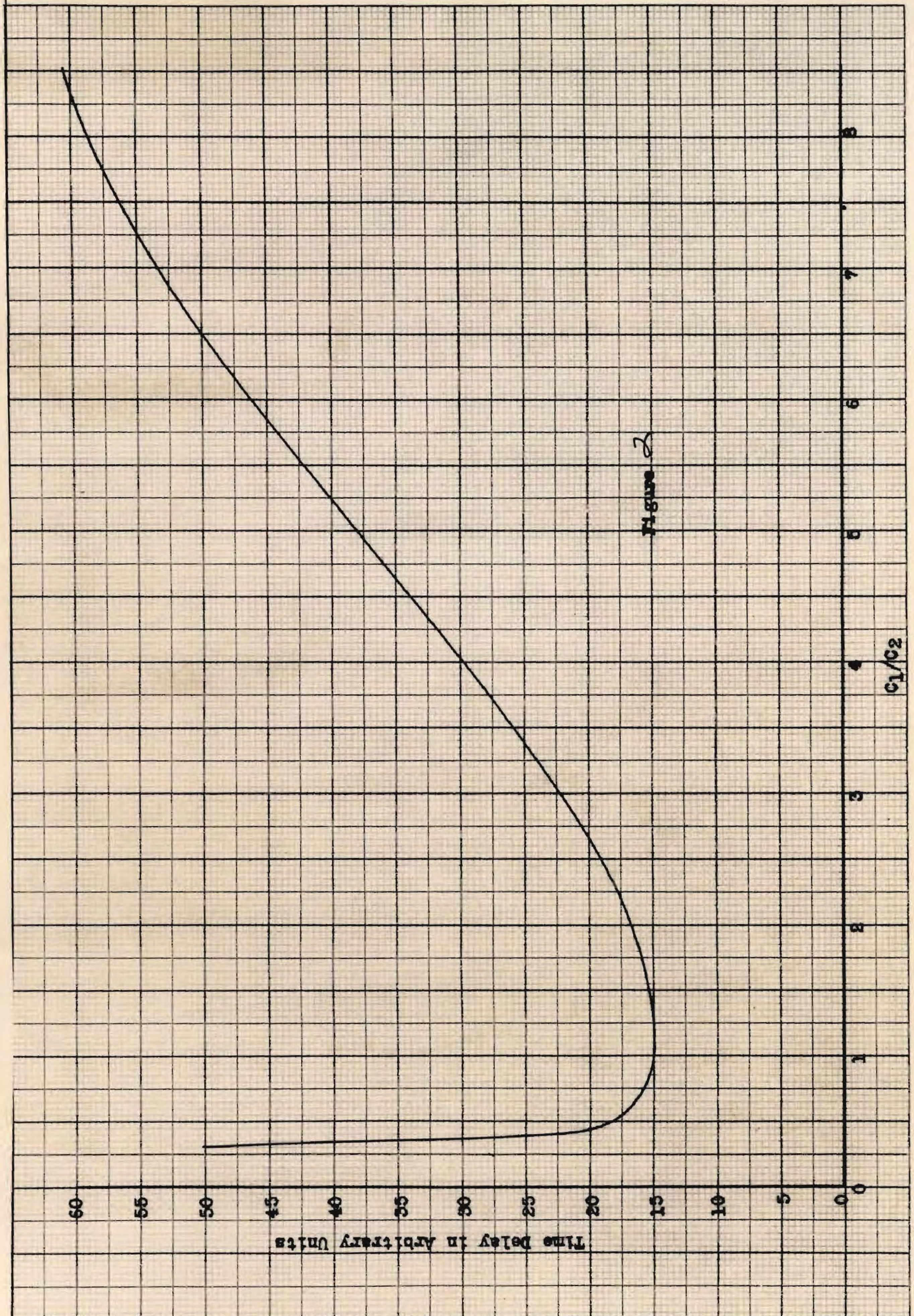


Figure 2
C_1/C_2
Time Delay in Arbitrary Units
0
1
2
3
4
5
6
7
8
0
5
10
15
20
25
30
35
40
45
50
55
60

varied from .5 to 2 without appreciably affecting the rise time.

C_1	C_2	C_1/C_2	Time Delay
30	150	.33	50 mm.
40	70	.57	17 mm.
50	50	1.00	15 mm.
70	40	1.75	16 mm.
100	30	3.33	23 mm.
150	30	5.00	38 mm.
250	30	8.33	60 mm.
500	25	20.00	No oscillations

Table 2.
Correlation of Time Delay with Ratio C_1/C_2.

Variation of grid return resistor.

The effect of varying the grid return resistor, R, was next determined. R was varied from a point below which no oscillations could be obtained up beyond the point at which self-blocking occurred, and measurements were made of the rise time and the amplitude of oscillation. The data obtained are presented in Figures 3 and 4 which show respectively the rise time and amplitude of oscillations as a function of R. It was observed that for very low values of R (R = 10,000 ohms) oscillations were very unstable and the rise time was very long. Depending upon the crystal, stable oscillations were obtained for R in the range, 20,000 to 50,000 ohms. Under this condition the amplitude of oscillation was large but the rise time was still long. As R was increased, the rise time decreased, but the amplitude also decreased. Finally, depending on the crystal, self-blocking occurred for R greater than about 20 megohms.

A qualitative explanation for this behavior is offered. For low values of R the losses in the circuit are so great that oscillations cannot exist. As R

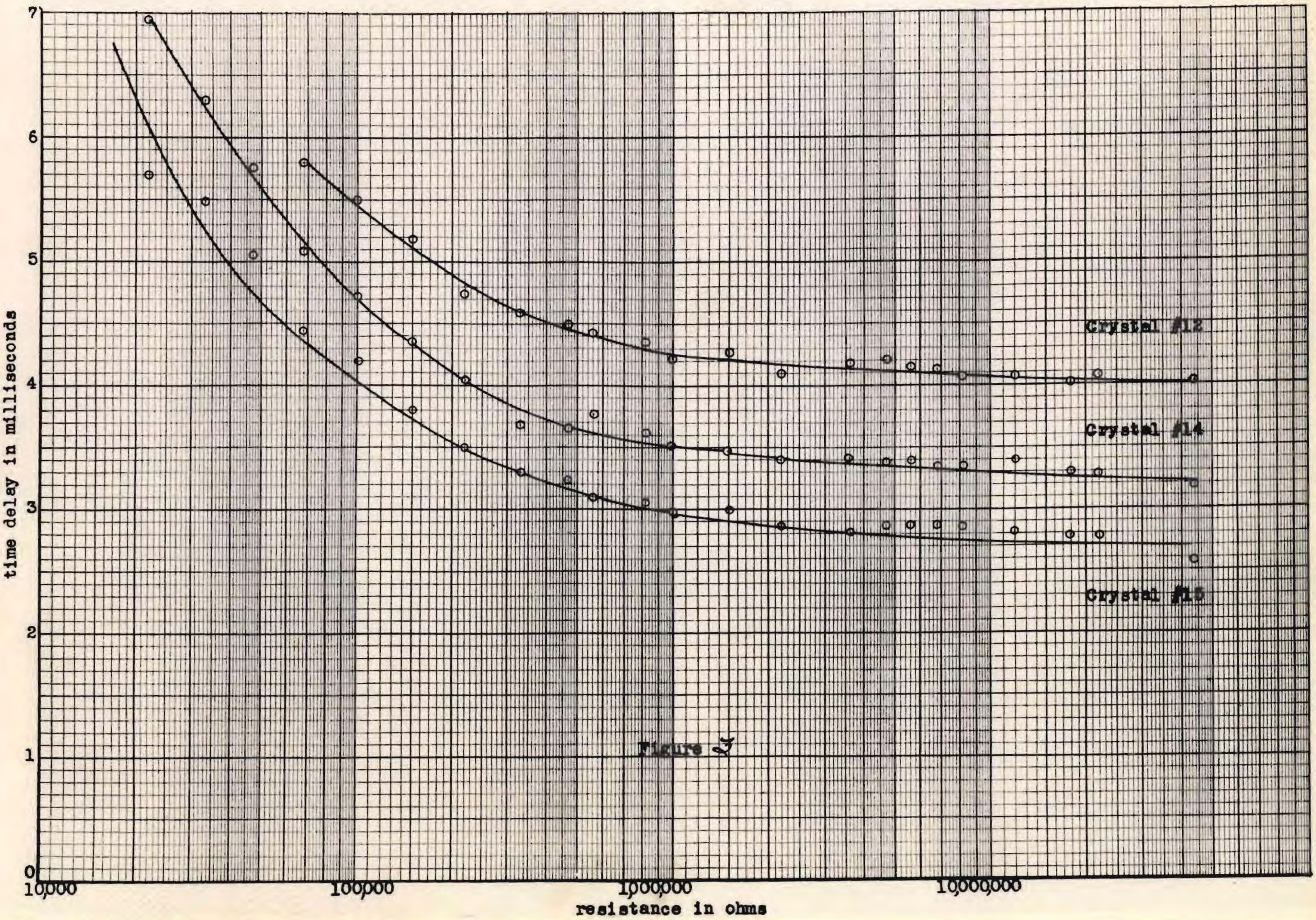

time delay in milliseconds
0
1
2
3
4
5
6
7
10,000
100,000
1,000,000
10,000,000
resistance in ohms
Crystal #12
Crystal #14
Crystal #15

Figure 3

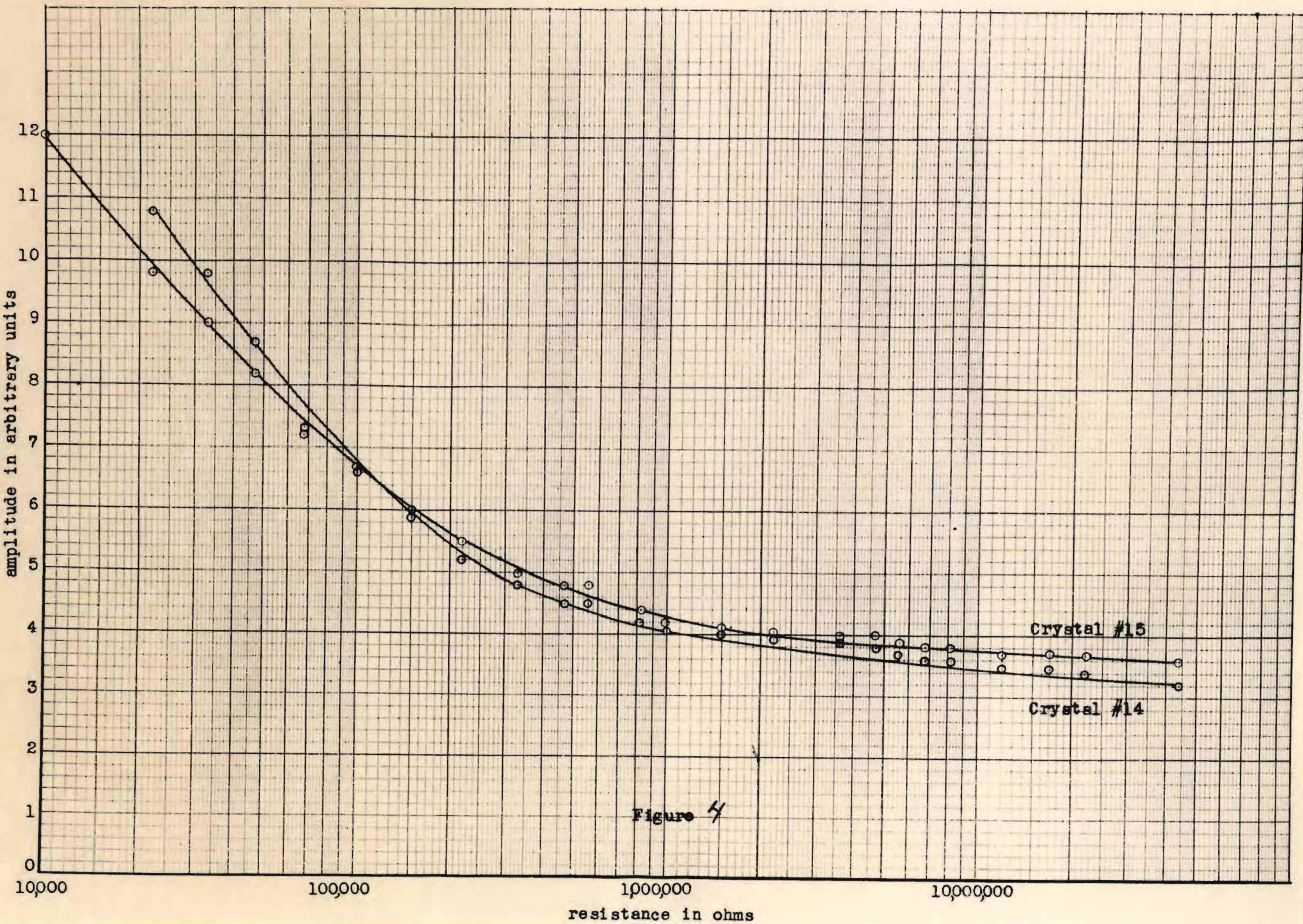

amplitude in arbitrary units
12
11
10
9
8
7
6
5
4
3
2
1
0
Crystal #15
Crystal #14
10,000
100,000
1,000,000
10,000,000
resistance in ohms

Figure 4

increases these losses decrease, but there is still not sufficient gain left over (after the losses are cancelled) to build up oscillations at a rapid rate. This will account for the long rise time. To determine the amplitude effects, consider the self-biasing operation in an oscillator. The grid draws a small amount of current at the peak of each swing. This charges the capacity in the grid circuit to produce a negative D.C. bias. During the time the grid is not drawing current, some of this charge leaks off the capacity through R. The next positive half cycle the grid draws enough current to replace this lost charge. Now, when R is low, a rather large portion of this charge leaks off when the grid is not conducting. It is then necessary for the grid to draw a relatively larger current to replace this lost charge; to do this the grid must go considerably positive with respect to the cathode. This causes a large plate current which in turn makes the plate voltage drop to a low value. It can be seen that increasing R decreases the grid circuit loss and thus requires a smaller grid swing and, therefore, a smaller plate swing.

From this limited experimental data, it appears reasonable to conclude that R should be greater than 1 megohm for use in keyed oscillators. It is the opinion of the author that 4 megohms $\leq$ R $\leq$ 8 megohms represents a reasonable design limitation. There seems to be no justifiable reason for making R $>$ 8 megohms because larger values incur the risk of self-blocking. This topic will be further examined and will be discussed in the final report.

It can be shown that the losses the grid return resistor produces in an equivalent AC. circuit may be represented by replacing R by a resistance of value, $\frac{R}{3}$.[1] Thus it is seen that this information may be interpreted as the effect of loading the oscillator by an external load.

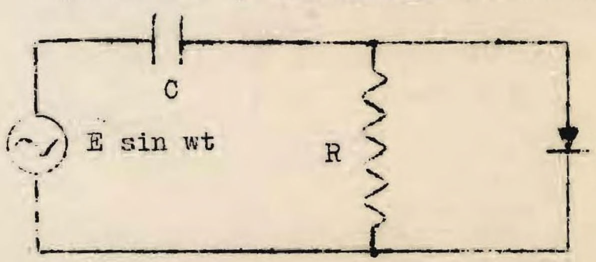
<u>Variation of plate load impedance.</u>

The next step was to determine the effects of varying the plate load impedance, L. The data obtained are tabulated in Tables 3 and 4.

L	Crystal #11	Crystal #12	Crystal #13	Crystal #14	Crystal #15
50.5 μ h.	1.1 ms.	1.2 ms.	1.2 ms.	1.3 ms.	1.3 ms.
221 μ h.	1.3 ms.	1.35 ms.	1.3 ms.	1.4 ms.	1.4 ms.
498 μ h.	1.3 ms.	1.4 ms.	1.35 ms.	1.55 ms.	1.45 ms.
750 μ h.	1.15 ms.	1.3 ms.	1.2 ms.	1.35 ms.	1.4 ms.
1.50 mh.	1.3 ms.	1.35 ms.	1.2 ms.	1.4 ms.	1.3 ms.
4.5 mh.	1.4 ms.	1.5 ms.	1.4 ms.	1.7 ms.	1.55 ms.

Table 3.
Correlation of Time Delay with Plate Load Impedance.

(1) This effect is due to grid circuit rectification. When grid rectification occurs, the load presented by the grid circuit to a generator may be represented by the circuit in the figure. The A.C. component of power due to C and R is:



$$P_{AC} = \frac{\hat{E}^2}{2R}$$

Now the condenser will assume a D.C. voltage due to rectification by the grid. If RC is large (and it usually is) compared to the period of w, this D.C. voltage will equal $\hat{E}$. Then there is a D.C. component of power

$$P_{DC} = \frac{\hat{E}^2}{R}.$$

The total power dissipated, $P = P_{AC} + P_{DC} = \frac{3}{2}\ \frac{\hat{E}^2}{R}$.

To obtain a single resistor which will dissipate the same amount of power as the above rectifying circuit, it is necessary to use a resistance equal to $\frac{R}{3}$.

Crystal	11	11	11	11	11
L	50.5 μ h.	221 μ h.	498 μ h.	750 μ h.	1.5 mh.
Amplitude	10 div.	7.6 div.	7.4 div.	6.2 div.	7.0 div.

Table 4.
Correlation of Oscillation Amplitude with Plate Load Impedance.

It is noted that the value of L appears to have little effect on either R.T. or amplitude of oscillation as long as the resonant frequency of L with C_2 and distributed capacity is reasonably lower than the crystal frequency. As the resonant frequency of LC_2 approaches the crystal frequency, the R.T. will decrease and the amplitude of oscillations increase; however, if wide band operation is contemplated, operation with the resonant frequency of LC_2 near the crystal frequency would require some method that is a tuning control for changing L as the crystal frequency is changed. One of the chief advantages of the Pierce oscillator is the fact that it can be made to operate over a large range of frequencies without requiring any tuning controls. It is believed that any advantage resulting from a decrease in rise time and increase in amplitude, obtained through the use of a tuning control, would not equal the disadvantage incurred by the resultant reduction in flexibility. For this reason it seems that L should be of such a value that the resonant frequency of LC_2 would be considerably lower than the lowest crystal frequency contemplated.

Variation of tube parameters.

The final circuit parameter to be varied is the vacuum tube. The two tube characteristics of interest are the r_p and g_m. Because the r_p may be interpreted as equivalent to the grid return resistor, R, insofar as its effects are concerned, the effect of varying r_p is known.

To study the effect the g_m has on rise time requires the use of a 6L7 in the special circuit arrangement shown in Figure 5.

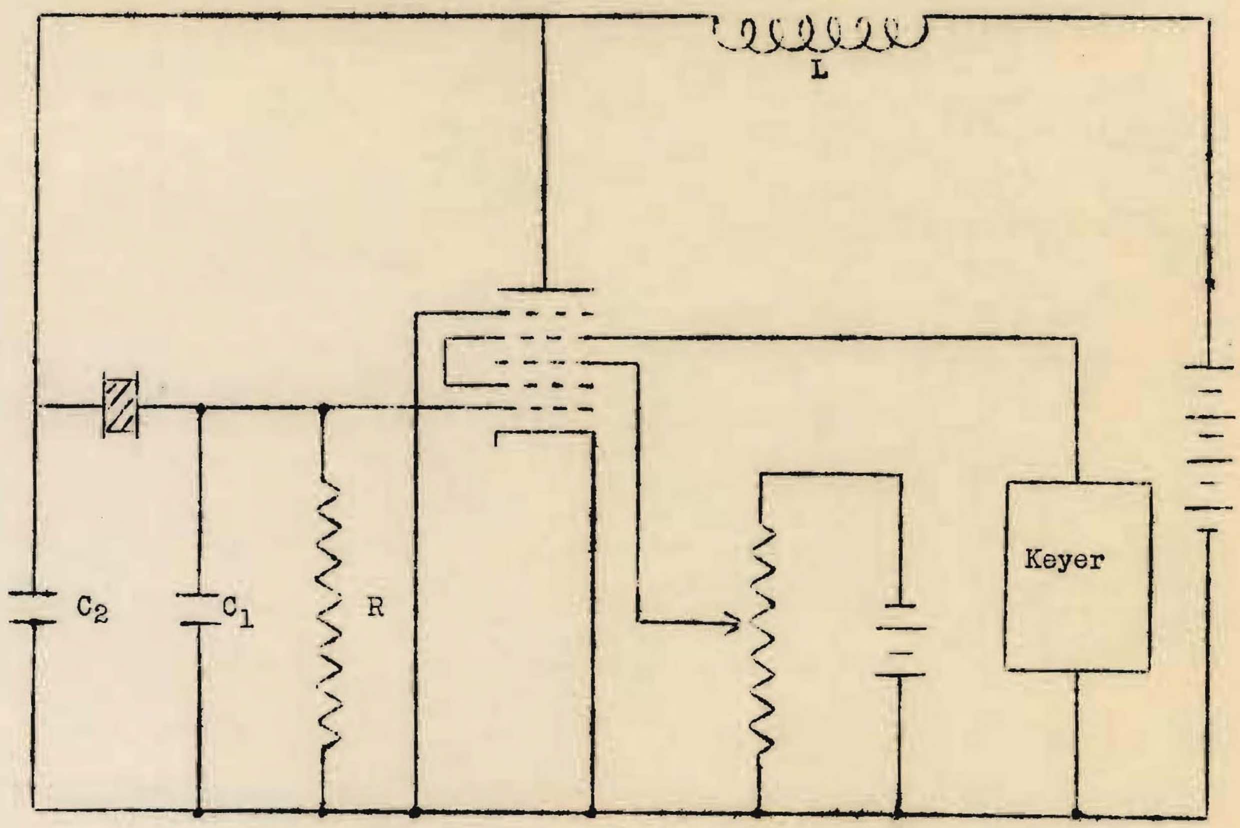


Figure 5.
Oscillator Circuit Using Variable g_m Tube.

By changing the bias applied to the third grid, it is possible to vary the transconductance between the first grid and the plate. This characteristic is shown in Figure 6. The R.T. was measured for various values of bias on G_3; these data are tabulated in Table 5.

It is seen that the R.T. increases as the g_m decreases. This recommends the use of high g_m tubes to promote a short R.T.

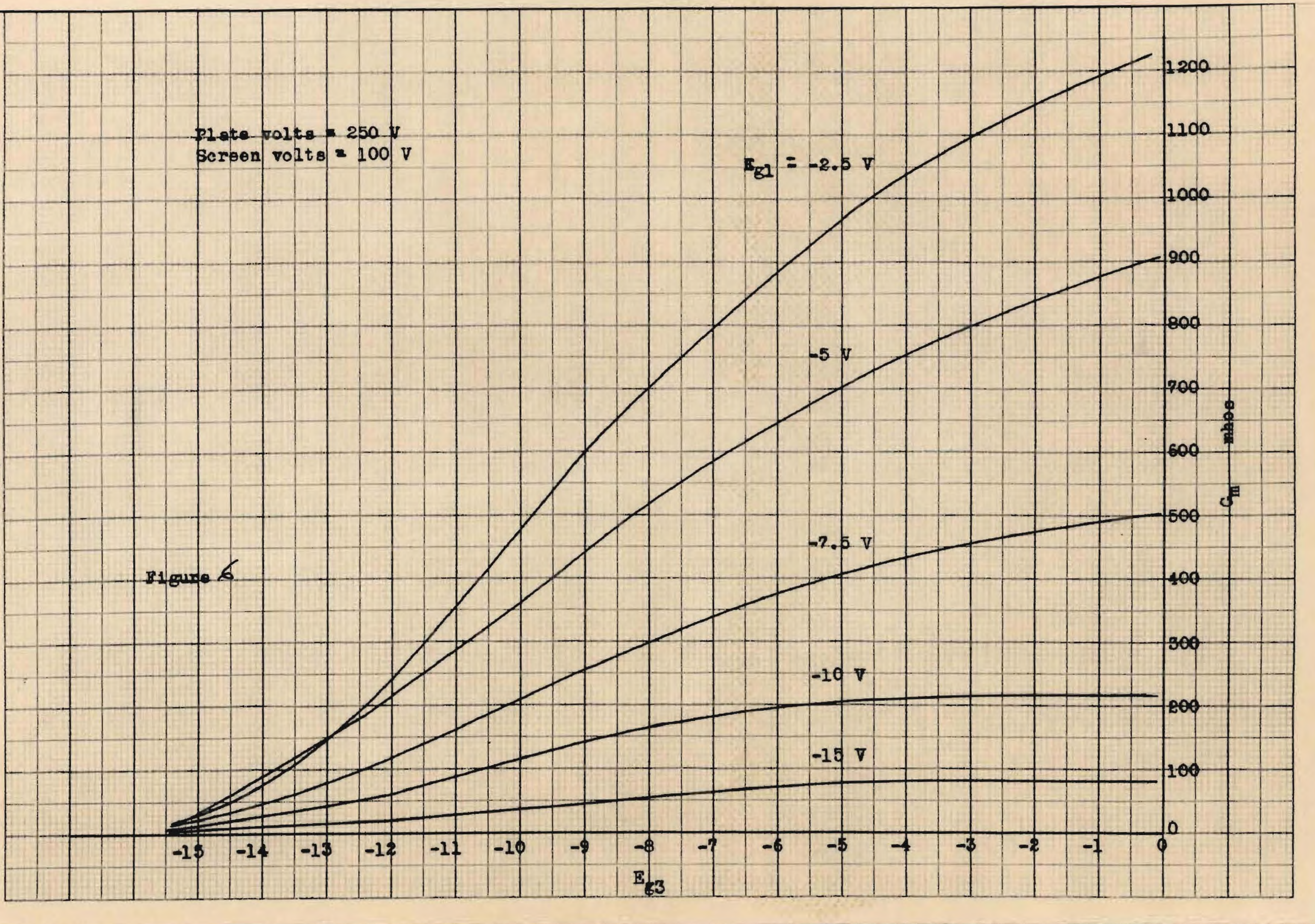


Plate volts = 250 V
Screen volts = 100 V
E_{g1} = -2.5 V
-5 V
-7.5 V
-10 V
-15 V
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
G_m mhos
-15
-14
-13
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
E_{g3}

Figure 6

E_{G_3}	Rise Time
0	35
-2	33
-4	53
-6	69
-8	92

Note: The time delay is measured in arbitrary units. It was found that oscillations ceased when E_{G_3} was more negative than -8.5 volts. Plate current became zero when E_{G_3} was more negative than -18 volts.

Table 5.
Correlation of Tube g_m with Time Delay.

Summary of preliminary observations.

It was found that:

(a) High Q crystals produce long R.T.'s and F.T.'s.

(b) When the circuit parameters meet other restrictions, the Q of the crystal determines the F.T.

(c) The ratio C_1/C_2 may vary between 1/2 and 2 without appreciably affecting the R.T. A value, $C_1/C_2 = 1$, produces minimum R.T.

(d) R should be large, preferably in the range of 4 to 8 megohms.

(e) L should have a value such that $wL \gg \frac{1}{wC_2}$ at the crystal frequency.

(f) Vacuum tubes having large values of r_p and g_m should be used.

Referring to Table 1, it is apparent that the F.T. is considerably longer than the R.T. This suggests the possibility that the fall time is the limiting factor in high speed keying.

II. OPTIMUM WAVEFORMS FOR KEYING

Since the preceding experiments indicated that the rise time is much shorter than the fall time, the question arises concerning the type of keying waveform (or envelope) which is most desirable. In the conference held at Long Branch in December this same question was discussed, and it developed that there is no existing criterion by which to determine the optimum waveform of a keyed oscillator. Accordingly, work was undertaken to determine those properties of a keyed wave which affect its usefulness. The broad objective which governed this work was to obtain maximum intelligibility of signals in conjunction with a minimum use of the frequency spectrum.

Because a keyed signal will be judged by the human ear (excluding machine recorders) and because the behavior of the ear is not readily expressed in mathematical terms, the study was conducted on a purely experimental basis. Signals of various forms were applied to the terminals of a loudspeaker and the resulting signal was judged by several observers. The actual waveform was recorded in each case by means of an oscilloscope. The methods used and the results obtained are described in the following paragraphs.

General procedure.

Experimentally, a successful examination of this problem requires that some method be provided to obtain an audio frequency modulated with various keying waveforms. This objective is illustrated in Figure 7.

Various methods were tried to produce this effect, the unsuccessful or unpromising ones being included at the end of this section. The final experimental setup is shown in Figure 8. Circuit diagrams for the keyed oscillator and converter are shown in Figures 9 and 10 respectively. The operation of

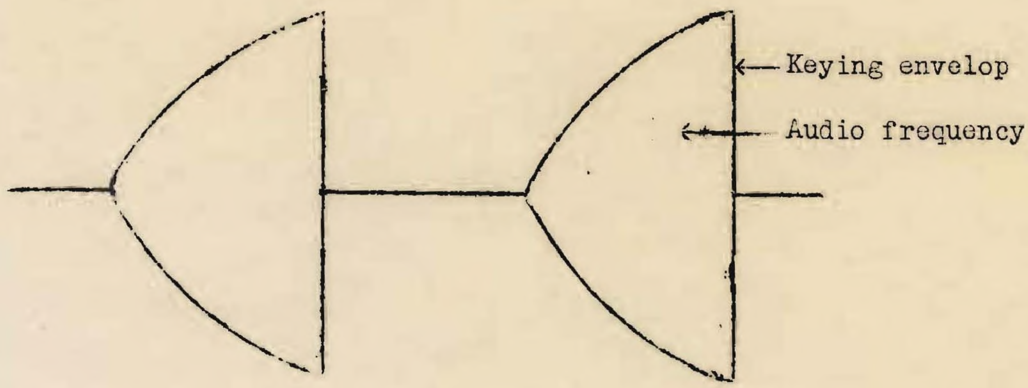


Figure 7.
Keying Waveform.

the setup is relatively simple. The output of the keyed oscillator is mixed in the mixer stage with the output of a crystal controlled cw oscillator which has an output frequency differing from the keyed oscillator by a few hundred cycles. Provision is made for varying the frequency of the cw oscillator to obtain a pleasing tone. Since the mixer is a nonlinear device, there exists in the plate circuit an audio frequency which is the difference of the two radio frequencies. This audio frequency is passed through filters to remove any RF present and then amplified to drive a loudspeaker. Oscillograms are made of the voltage waveforms at the keyed oscillator plate and across the speaker terminals.

Observations and data.

Oscilloscope photographs were taken at points (a) and (b) (Figure 8) for various keying waveforms. These photographs with comments are presented in Figure 11. These comments are the result of averaging the opinions of several persons (acquainted with code transmission) who listened to the signals.

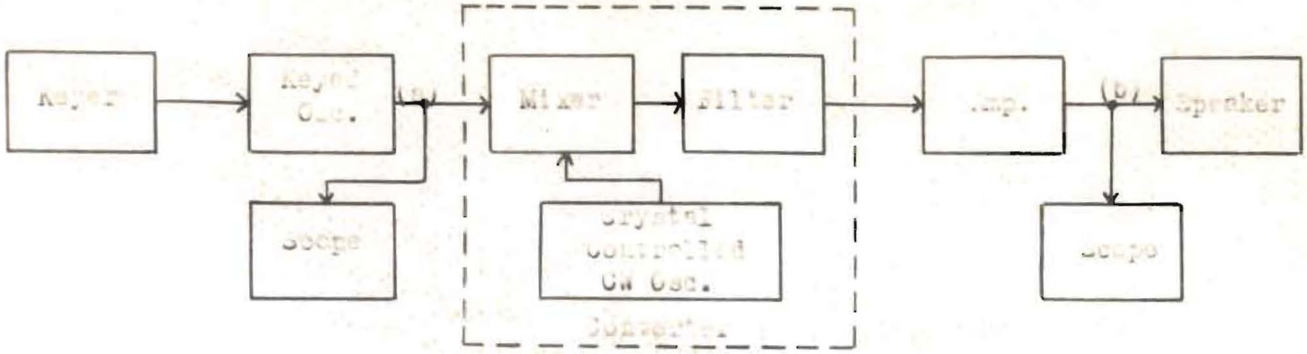
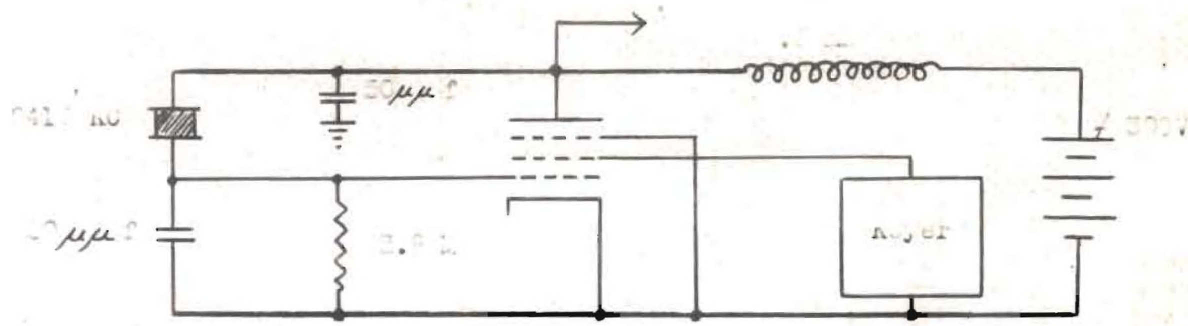
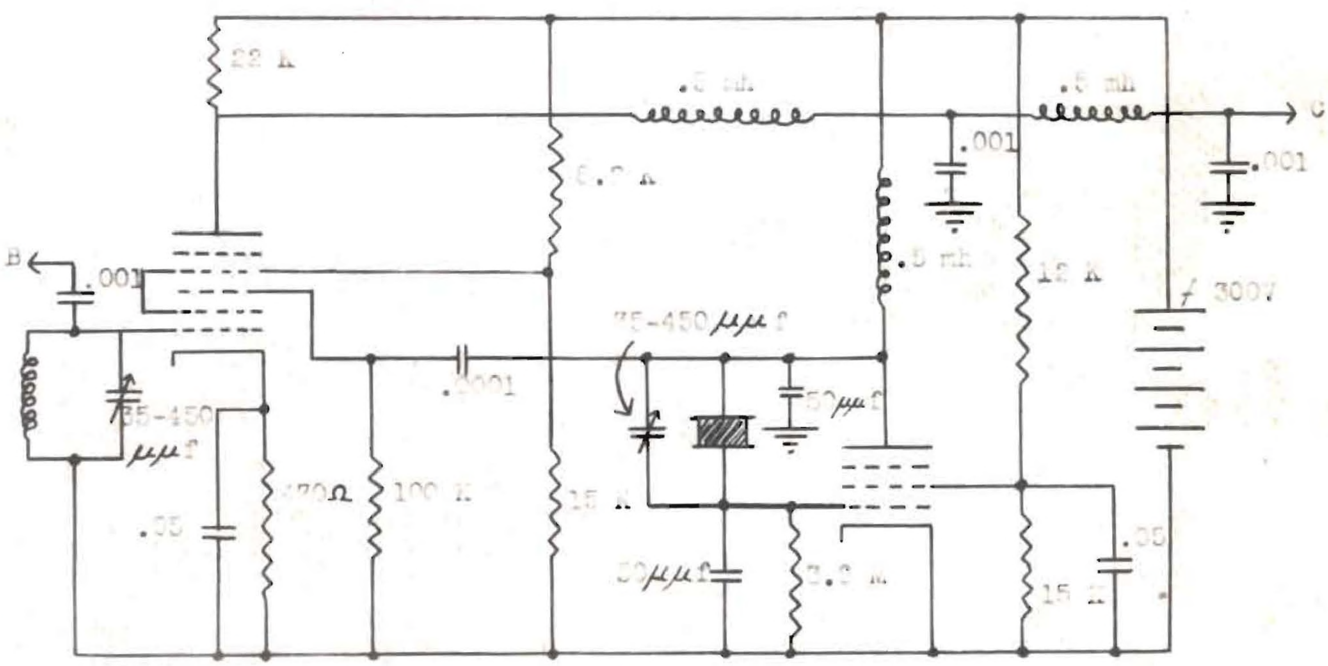


Figure 8



A - Connected direct to Y-axis input on oscilloscope and having loose capacitive coupling to the mixer.

Figure 9

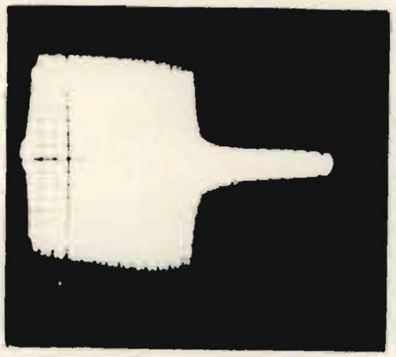


B - Loosely coupled to keyed oscillator
C - To input of audio amplifier

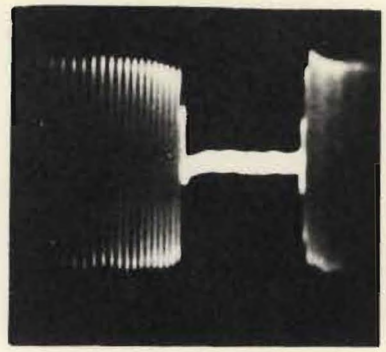
Figure 10

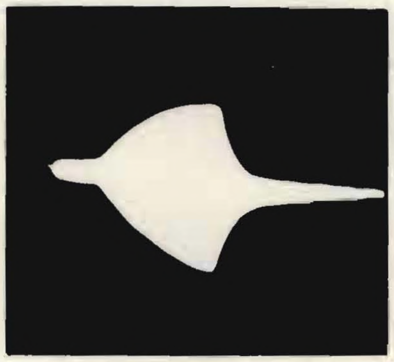
Oscillator Plate (a)

Speaker Terminals (b)

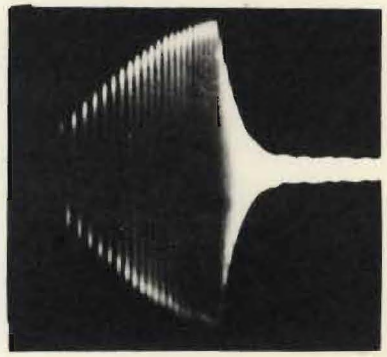


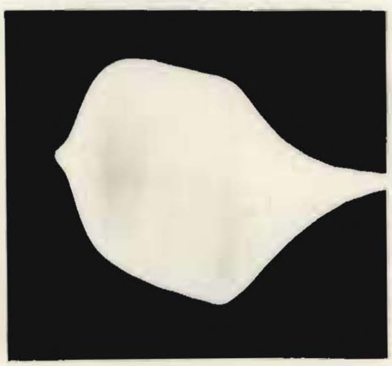
Square wave keying
Perfectly readable
Noticeable key clicks



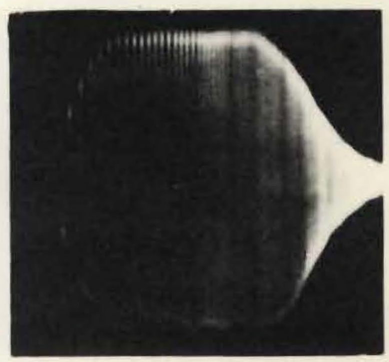


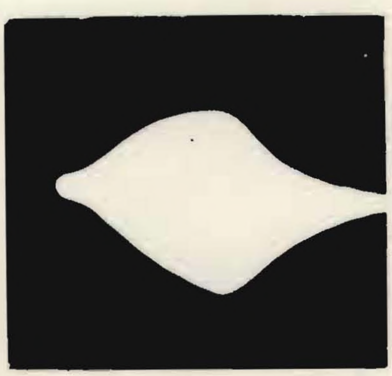
Long rise time
Short fall time
Readable
No noticeable key clicks





Medium rise time
Long fall time
Readable with difficulty
No noticeable key clicks





Long rise time
Long fall time
Unreadable
No key clicks

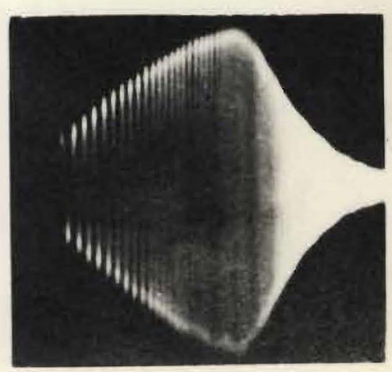


Figure 11.

The following qualitative results were obtained:

(a) Essentially square wave keying gives a perfectly readable signal, but key clicks are produced; however, the majority of the observers expressed no objection to slight clicks, and a few expressed a preference for slight key clicks.

(b) A relatively long rise time does not degrade the readability of a signal; moreover, most of the noticeable clicks disappear.

(c) A relatively long fall time definitely degrades the readability.

(d) A long rise time and a long fall time produce unreadable signals.

(e) The most noticeable clicks are produced by a short rise time. The clicks produced by a short fall time are relatively unimportant.

The following conclusions are drawn:

(a) The fall time must be small compared to the period of one dot-cycle.

(b) The rise time should not be too short or objectionable clicks will be produced. A relatively long rise time will still produce readable signals.

Tentative recommendations.

For the study of keyed crystal controlled oscillators it was decided arbitrarily to assign maximum values in per cent of the period of a dot-cycle for the rise and fall times. In view of the above conclusions it is believed that for adequate performance:

(a) The time for the keying waveform to reach 80% of maximum value should not exceed 25% of the period of one dot-cycle.

(b) The time for the keying waveform to decrease to 20% of maximum value should not exceed 10% of the period of one dot-cycle.

(c) The rise and fall curves should approximate $(1 - e^{-kt})$ and (e^{-kt}) respectively.

This limiting condition is shown in Figure 12. It is emphasized that these values are arbitrary, but it is believed that they will produce satisfactory operation.

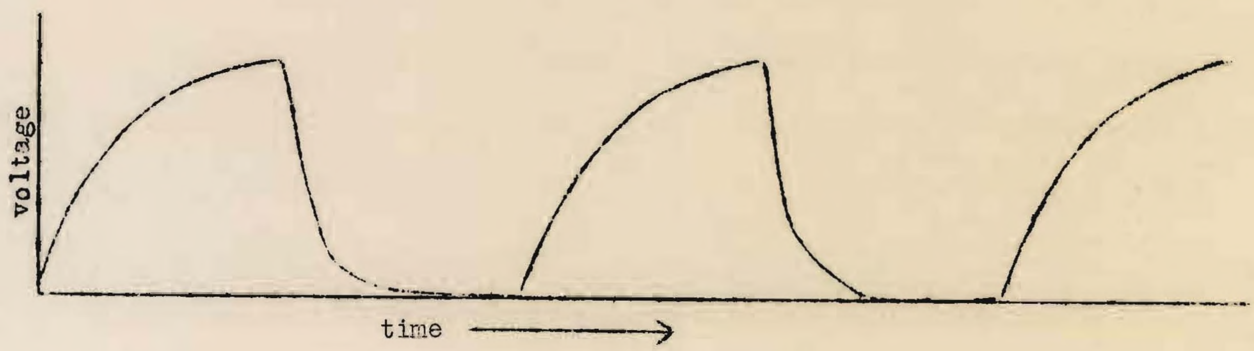


Figure 12.

To apply these results to crystal damping, it is desirable to interpret them in terms of $\triangle$, the damping decrement. A specific temporary goal of 43 dot-cycles per second has been given for keying oscillators.[1] Then

$$\triangle = 20 \log_{10}\frac{E_2}{E_1} \; \frac{1}{\text{period x 10\%}} \; \text{db/sec., and} \qquad (1)$$

$$\triangle = 20 \log_{10} \cdot \frac{2E_1}{E_1} \cdot \frac{1}{\frac{1}{43} \times .1} \; \text{db/sec.} = 6011 \text{ db/sec.} \qquad (2)$$

A suitable and convenient figure will be

$$\triangle = 6000 \text{ db/sec.} = 6 \text{ db/millisecond.} \qquad (3)$$

Thus it is seen that a $\triangle \geq 6$ db/ms is required for an oscillator to key

(1) Conference at Long Branch Frequency Control Laboratory, December 10 and 11, 1946.

properly at 43 dot-cycles per second. This establishes a value for the fall time. The rise time may not be expressed so simply because it usually involves two or more exponential terms having opposite curvature.

Direct gating of an audio oscillator.

Because the desired data on keying waveforms are basic in character, it seems desirable to secure these data in as direct a manner as possible. In particular, it was hoped that the measurements could be secured without the use of radio-frequency keying so that a completely independent check would exist.

To this end an attempt was made to gate the output of an ordinary cw audio-frequency oscillator. A block diagram describing the method used for producing these waveforms and a circuit diagram of the mixer designed for the setup are shown in Figures 13 and 14, respectively. The mixer, which uses two 6L7's, is of the balanced gating type. It was so constructed that when voltage gates of equal amplitude but of opposite phase are applied to the two grids, a constant current flows through the common load resistance. The gate amplitude was made large enough so that when one tube is conducting, the other tube is cut off. Therefore, the audio signal which is applied to one tube passes through only when that tube is conducting, but is immediately cut off when the tube is cut off. It was believed that by the proper biasing of the 6L7's the nonlinearity of the tube could be cancelled and the desired effects produced. However, experimentally this was not found to be the case. Only with a square wave input was the anticipated output waveform produced. The operation is shown in Figure 15.

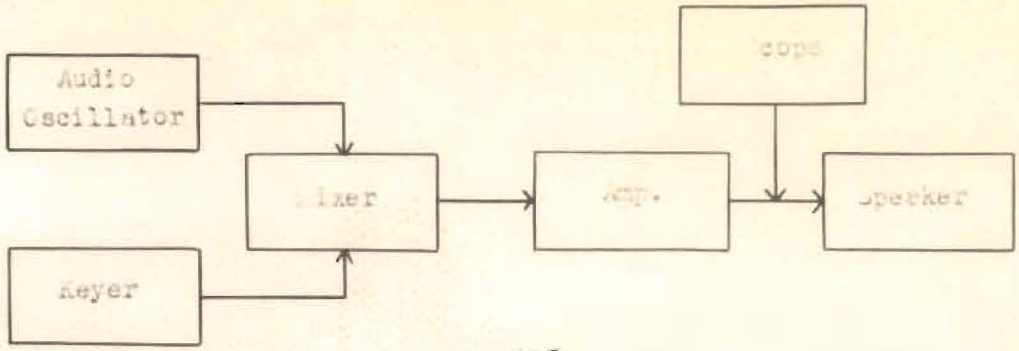


Figure 13

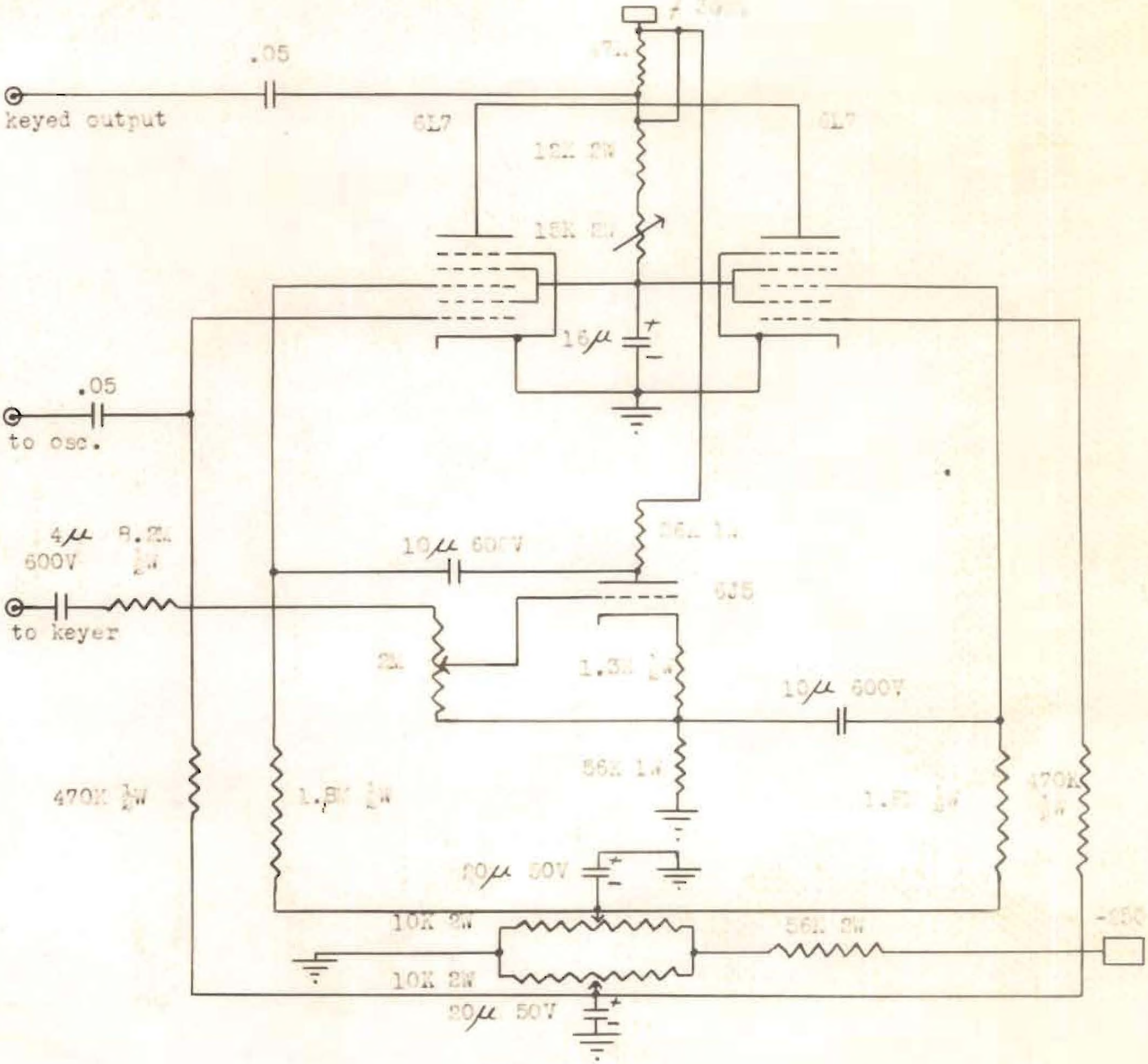
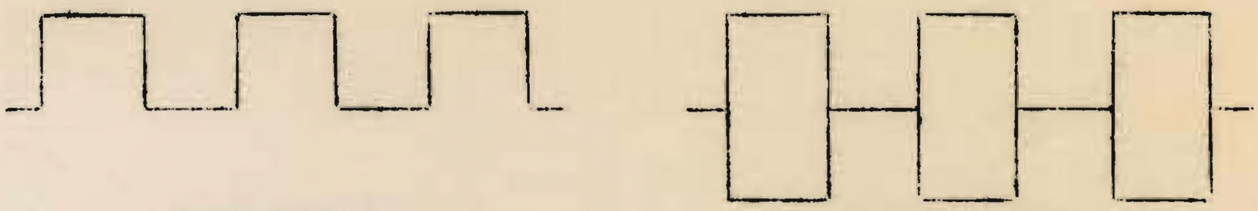


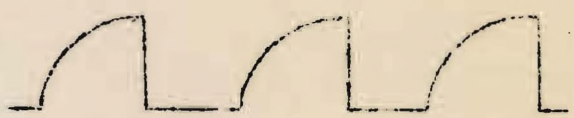
Figure 14



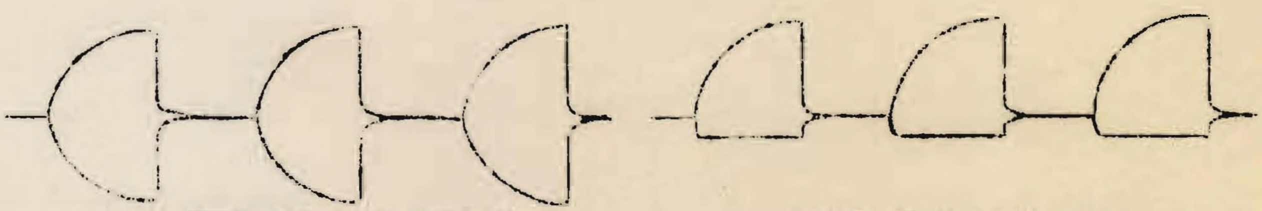
Input Waveform Output Waveform

Figure 15.
Keying Waveform for Square Wave Control.

Other input waveforms with their anticipated and actual output waveforms are shown in Figures 16 and 17.



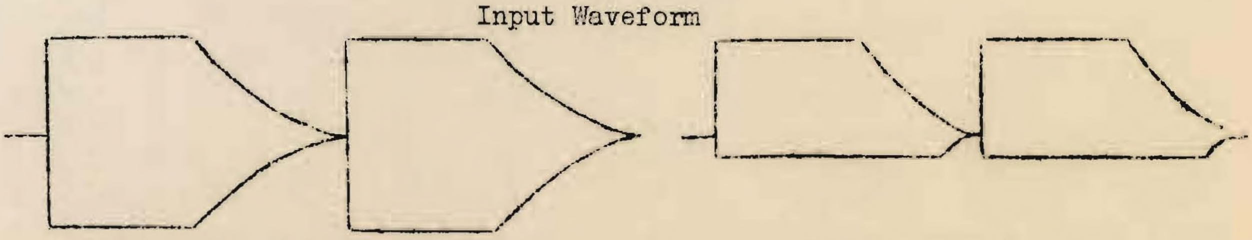
Input Waveform



Anticipated Output Waveform Actual Output Waveform

Figure 16.
Keying Waveform for Long Rise Time.

Input Waveform



Anticipated Output Waveform Actual Output Waveform

Figure 17.
Keying Waveform for Long Fall Time.

Although the various tube biases could be varied independently, it was found impossible to make the actual output waveform the same as the desired waveform; therefore, this method was abandoned.

The basic difficulty is to obtain a variable transmission of the desired signal unaccompanied by a direct current pulse of equal or greater magnitude. The method described attempts to balance out this component of direct current by the use of a bridge circuit. The failure is due to the fact that the transconductance curve was neither straight nor symmetrical about its midpoint.

Use of conventional receiver.

Another method used in an attempt to produce the several waveforms desired is shown in Figure 18.

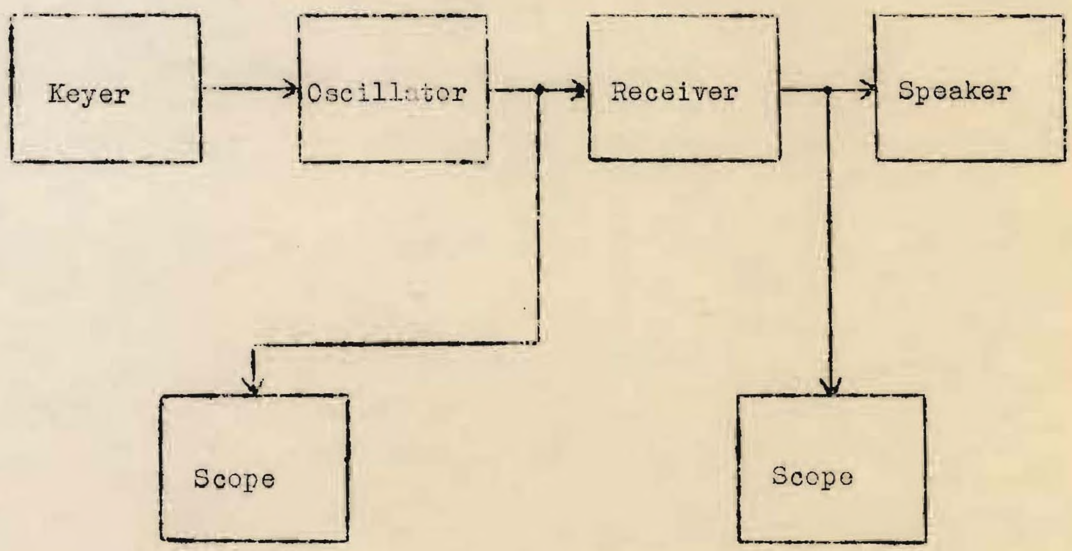


Figure 18.
Block Diagram for Circuit Using Receiver.

It was found that the effect of varying the waveform could be determined, but that the receiver seriously distorted the keying waveform. Because there was no way to evaluate the effects of the receiver distortion, this method was discarded. It should be noted that a radio receiver does introduce keying waveform distortion, particularly if a very sensitive and selective receiver is used. However, the effect of this distortion on readability did not seem to be appreciable.

An interesting implication of this work is that the receiver used, and probably many receivers of present design, produces appreciable distortion of the keyed waveform. For slow keying, this is probably unimportant, but for fast keying it may be necessary to employ specially designed or adjusted receivers.

III. DAMPING OF QUARTZ CRYSTALS IN OSCILLATOR CIRCUITS

The experiments described in Section I of this report show that the fall time of a crystal controlled oscillator is likely to be considerably longer than the rise time. This is explained by the fact that the rise time may be shortened considerably by the use of high gain tubes.

The keying tests described in Section II indicate that the fall time must be made considerably shorter than the rise time for optimum results.

These contradictory requirements can be reconciled by artificially shortening the fall time of the circuit. The Q of the circuit must be maintained at its normal high value in the "key-down" position in order to retain the full advantage of its frequency stability. But in the "key-up" position it is legitimate to damp the crystal vibration as rapidly as possible.

Pure resistance damping.

The simplest and most obvious method of damping a quartz crystal is by the connection of a resistance across its terminals. The actual and equivalent circuits for this method are shown in Figure 19. In practical circuits, it is necessary to provide a method (SW) of connecting R when the key is up and disconnecting R when the key is down. Since damping is desired only when the key is up, SW is considered closed for the purposes of this examination.

Define[1] $$X = \frac{1}{wC} \qquad X_z = \frac{1}{wC_z} \qquad X_o = \frac{1}{wC_o} \qquad n = \frac{C}{C_o} \tag{4}$$

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(1) For convenience in typing this report w is used throughout for the radian frequency, omega.

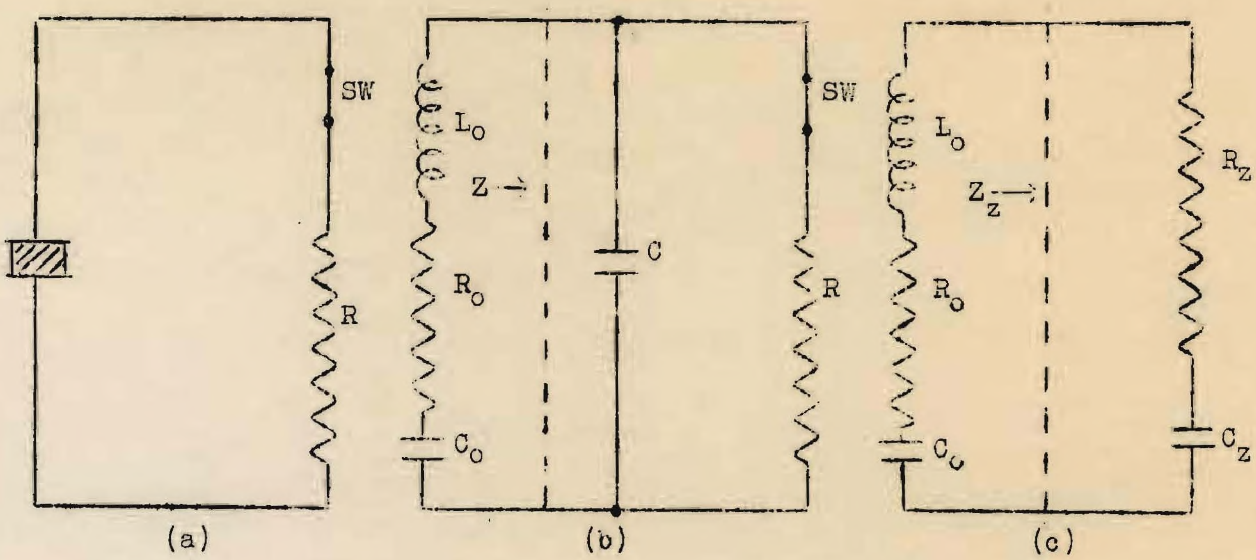


Figure 19.
Circuit for Damping the Vibration of a Crystal.

Equate the actual impedance, Z, to its equivalent impedance, Z_z.[1]

Then $$R_z - jX_z = \frac{-jXR}{R - jX} = \frac{X^2R - jR^2X}{R^2 + X^2} \tag{5}$$

Equating the real and imaginary components of Z and Z_z

$$R_z = \frac{X^2R}{R^2 + X^2} \qquad X_z = \frac{R^2X}{R^2 + X^2} \tag{6}$$

Now R_z is a maximum with respect to R when $\frac{dR_z}{dR} = 0$ (7)

Then $$\frac{dR_z}{dR} = \frac{X^2(R^2 + X^2) - 2R(X^2R)}{(R^2 + X^2)^2} = 0 \tag{8}$$

which requires that $R = X$ (9)

(1) This transformation is valid if the frequency is not changed appreciably in the process of making the transformation. It is known that a large variation in C does not change the frequency noticeably. Since the Q of the circuit is large even though R is added, R has little effect on frequency. Because of these facts it is reasonable to assume that the transformation could not affect the frequency enough to cause serious error.

The maximum R_z is given by

$$R_z = \frac{X^3}{2X^2} = \frac{X}{2} = \frac{R}{2} \tag{10}$$

Since $R_z \gg R_o$, the resonant Q of the circuit is

$$Q = \frac{X_o}{R_z} = \frac{2X_o}{X} = 2n \tag{11}$$

Decrement values.

Thus, it is seen that the minimum Q obtainable is equal numerically to twice the capacitance ratio C/C_o. Now the damping decrement, Δ, will be used to determine the effectiveness of this method.

$$\Delta = \frac{w}{2Q} \frac{\text{nepers}}{\text{sec}} = \frac{4.34w}{Q} \text{ db/sec} = \frac{4.34w}{2n} \text{ db/sec} = \frac{27.3f}{2n} \text{ db/sec.} \tag{12}$$

In this case the damping rate increases as the frequency increases and decreases as n increases. Since operations will be conducted over a considerable range of frequencies, it is encouraging to note that the higher frequencies promote rapid damping. Equation 12 recommends the use of as small a capacitance ratio as possible.

It has already been shown that Δ must be greater than 6 db/millisecond if minimum satisfactory damping is attained for 43 dot-cycles/second. In actual practice, n may be expected to vary between 250 and 2000. Assuming the larger value and $f = 1$ mcps., Δ is calculated as follows:

$$\Delta = \frac{27.3 \times 1 \times 10^6}{2 \times 2000} \text{ db/sec} = 6.83 \text{ db/millisecond.} \tag{13}$$

Thus, it is seen that satisfactory operation can be obtained by this method of damping.

Substitution of crystal.

For engineering application of this damping method, certain circuit and operational requirements must be considered. Crystals are designed to operate into a specified load capacitance, K; therefore, it is necessary to consider this capacity in the damping circuit.

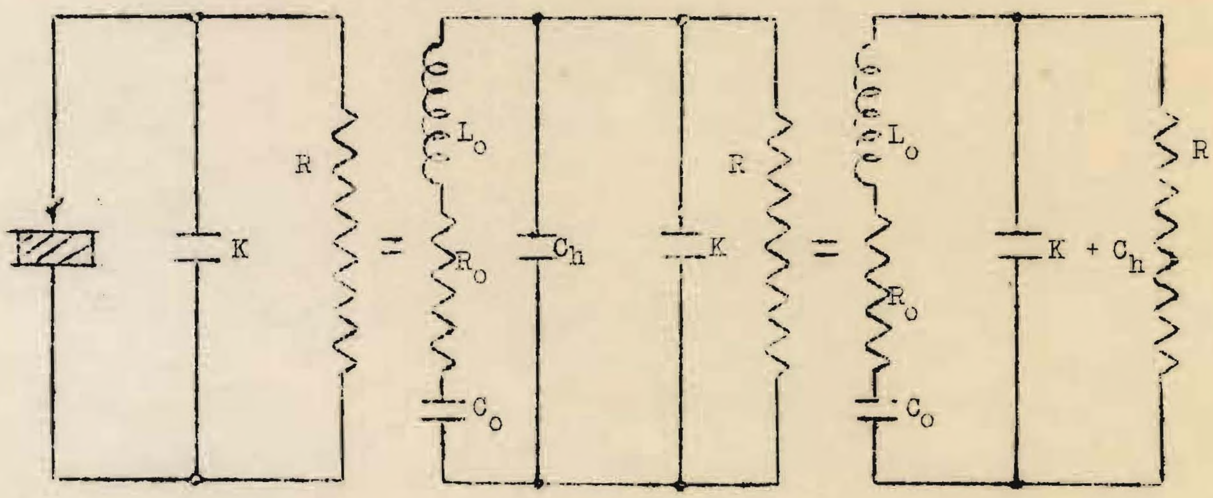


Figure 20.
Circuit Including Holder Capacitance.

Referring to Figures 19 and 20, let

$$a = \frac{C_h}{K} \qquad C = K + C_h = K(1 + a). \tag{14}$$

Then

$$n = \frac{K(1 + a)}{C_o}$$

and the R required for maximum damping is

$$R = \frac{1}{wC} = \frac{1}{wK(1 + a)} \tag{15}$$

So

$$R_z = \frac{1}{2wK(1 + a)} \tag{16}$$

For engineering application it is desirable to know what effect the variation of frequency has on R_z when R is selected for some specific frequency and is not varied as the frequency is varied.

Let f_o be the frequency specified in calculating R and let f be the actual frequency where $f = cf_o$ and $w = cw_o$. Then, from Equations 6 and 15 and substituting cw_o for w

we have
$$R = \frac{1}{w_o K(1 + a)} = H, \tag{17}$$

and
$$R_z = \frac{\left[\frac{1}{wK(1 + a)}\right]^2 \cdot \frac{1}{w_o K(1 + a)}}{\left[\frac{1}{w_o K(1 + a)}\right]^2 + \left[\frac{1}{wK(1 + a)}\right]^2} = \frac{\left[\frac{1}{w_o K(1 + a)}\right]^3 \cdot \frac{1}{c^2}}{\left[\frac{1}{w_o K(1 + a)}\right]^2 \left[1 + \frac{1}{c^2}\right]} \tag{18}$$

$$R_z = \frac{1}{w_o K(1 + a)} \cdot \frac{1}{1 + c^2} = H \cdot \frac{1}{1 + c^2} \tag{19}$$

If R were readjusted for optimum damping at each frequency, a modified equation for R_z would be obtained

$$R = \frac{1}{w_o c\, K(1 + a)} = \frac{H}{c} \tag{20}$$

And from Equations 10 and 20

$$R_z = \frac{R}{2} = \frac{H}{2c} \tag{21}$$

Equations 19 and 21 are plotted for various values of c in Figure 21.

From Figure 21 it can be seen that even though R is selected at one specific frequency, R_z does not deviate from the maximum R_z obtainable by more than 10% for a frequency change of 2.5 times. Because optimum damping is unlikely to be needed at all points in the band, it is probable that this

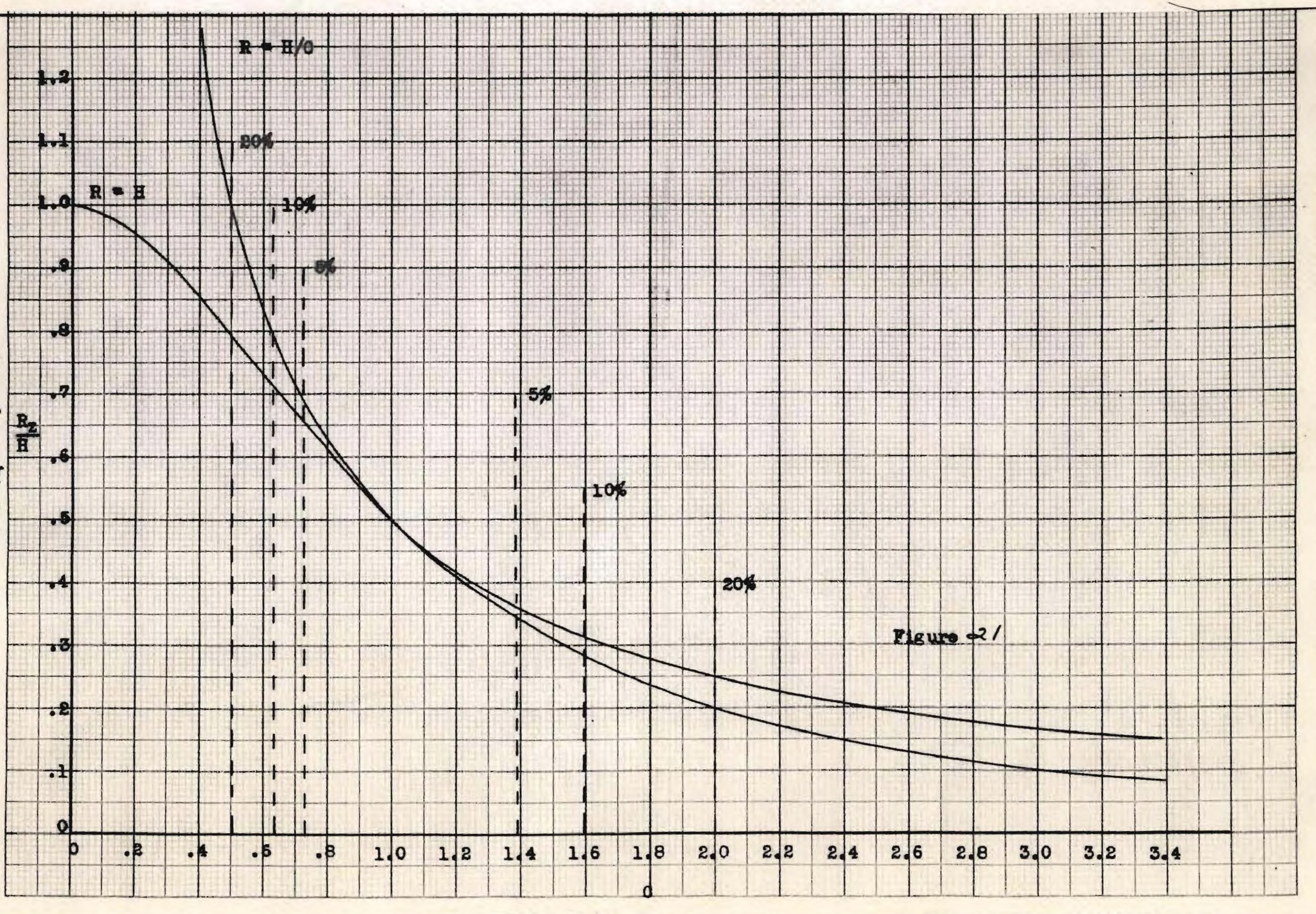

R = H/C
R = H
20%
10%
5%
5%
10%
20%
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
$\frac{R_Z}{H}$
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
C

Figure 21

ratio can be considerably extended. Thus, it is evident that satisfactory damping can be obtained over a reasonable frequency range with one specific damping resistance. Still larger frequency ranges may be covered by using several damping resistors with a selector switch.

Variation of holder capacitance.

It is necessary to consider the variation of crystal holder capacitance if operation with various types of crystals is contemplated. A variable, a, was introduced in Equation 14 where $a = \frac{C_h}{K}$.

It is estimated that C_h may vary between 5 mmf. and 25 mmf. A load capacitance of K = 32 mmf. has been tentatively specified as standard. Under these conditions a may vary between .156 and .781.

Let a_o be the value for which R is calculated. Then, from Equations 6 and 15 we have

$$R = \frac{1}{wK(1 + a_o)} = G \cdot \frac{1}{1 + a_o} \tag{22}$$

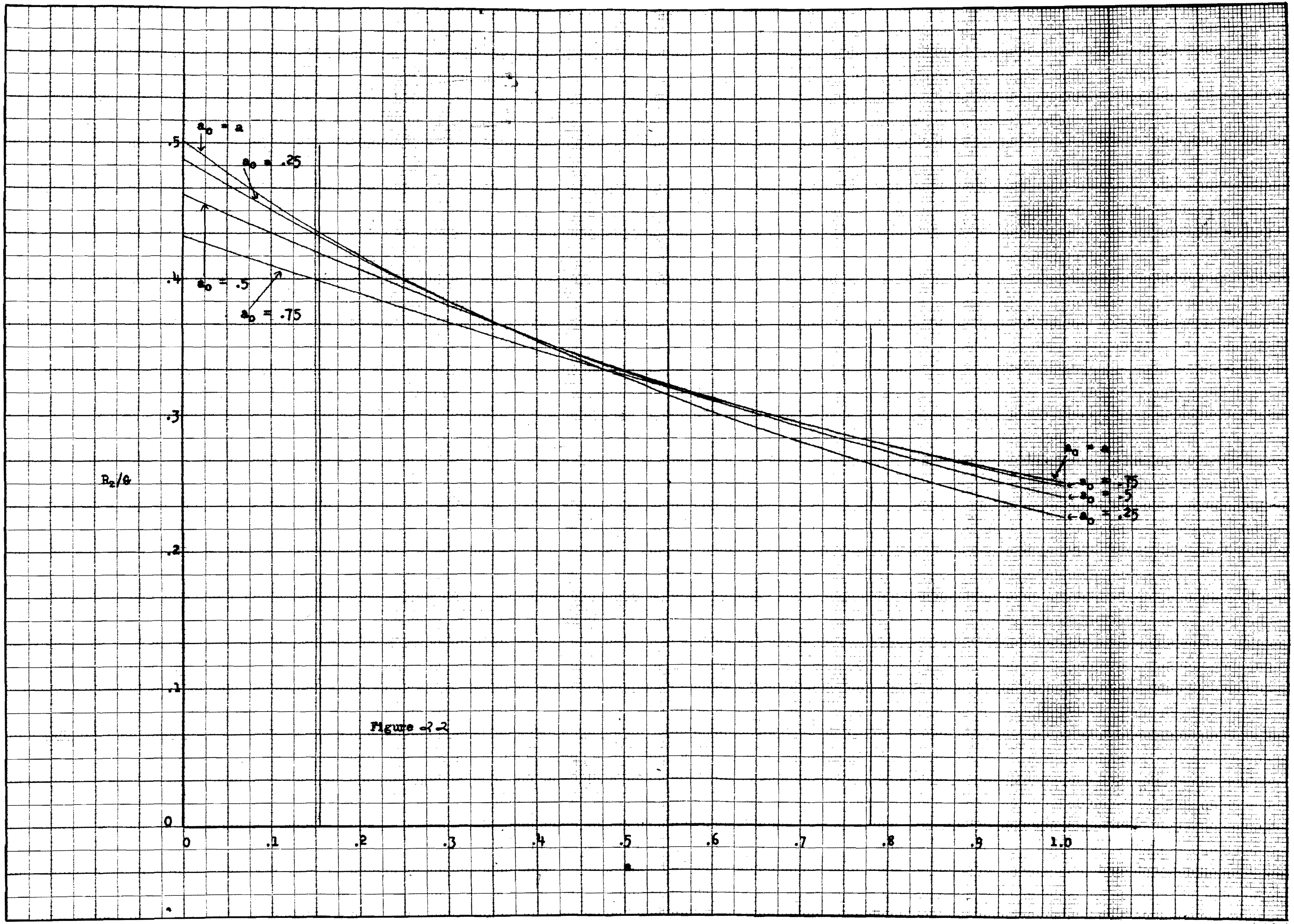
and

$$R_z = G \cdot \left[\frac{1 + a_o}{(1 + a_o)^2 + (1 + a)^2} \right] \tag{23}$$

where

$$G = \frac{1}{wK} \tag{24}$$

In Figure 22, Equation 23 is plotted for a_o = 0.25, 0.5, 0.75 and a, and for various values of a. The curve ($a = a_o$) represents the maximum damping obtainable (R is readjusted as a changes). From Figure 22 it can be seen that the damping obtained for a fixed value of R does not deviate appreciably from the maximum obtainable; specifically, for a_o = .5 the actual damping will not vary more than 3% from maximum even though a is varied from 0.16 to 0.78. Therefore, by a judicious selection of a_o the damping may be made essentially independent of crystal holder capacities encountered at present. It should be



$a_o = a$
$a_o = .25$
$a_o = .5$
$a_o = .75$
$a_o = a$
$a_o = .75$
$a_o = .5$
$a_o = .25$
.5
.4
.3
.2
.1
0
R_2/G
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
a

Figure 22

noted that as a increases, R_z decreases; therefore, larger holder capacities are unfavorable to rapid damping.

Frequency departures accompanying damping.

It is next necessary to determine the effect of damping upon the frequency of oscillation. Because of the inherent complexity of any analysis of oscillators and the approximations involved therein, it is elected to study the difference in frequency of a damped and undamped crystal and attempt to interpret this in terms of an actual oscillator circuit. It was stated in the footnote on page 28 that the frequency of oscillation is not changed appreciably in making the transformation shown in Figure 19b and 19c. This is correct in that the accuracy of the transformation is adequate for all practical purposes. It does not, however, mean that the exact frequency of the crystal is unaffected by the addition of the damping resistor R.

Consider then the two equivalent circuits shown in Figure 23, where R has the value to produce maximum R_z.

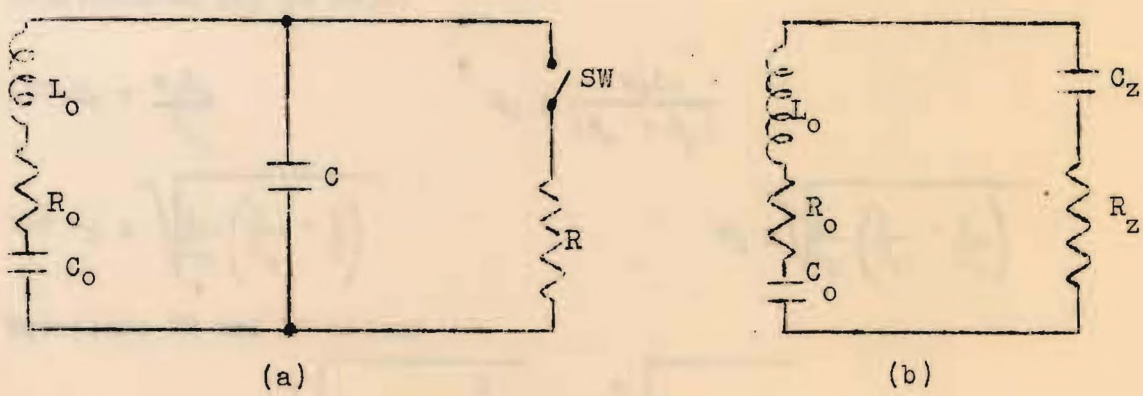


Figure 23.
Equivalent Circuits.

When the switch (SW) is closed, Figure 23a transforms to Figure 23b. From Equations 6 and 9

$$X_z = \frac{R^2 X}{R^2 + X^2} = \frac{X^3}{2X^2} = \frac{X}{2} \tag{25}$$

then $$C_z = 2C \tag{26}$$

and $$R_z = \frac{R}{2} = \frac{X}{2} \tag{27}$$

With the switch (SW) open, the frequency of the circuit in Figure 23a is given by

$$w_1 = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C}\right) - \frac{R_o^2}{4L_o^2}} \tag{28}$$

The frequency of the circuit in Figure 23b is given by

$$w_2 = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C_z}\right) - \frac{(R_o + R_z)^2}{4L_o^2}} \tag{29}$$

However, because both circuits have high Q's, certain approximations and simplifications may be made.

Let $Q_o = \frac{w_o L_o}{R_o}$ $\qquad Q_d = \frac{w_d L_o}{(R_o + R_z)}$ (30)

Let $w_o = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C}\right)}$ $\qquad w_d = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C_z}\right)}$ (31)

Equations 28 and 29 become

$$w_1 = \sqrt{w_o^2 - \frac{w_o^2}{4\,Q_o^2}} = w_o\sqrt{1 - \frac{1}{4\,Q_o^2}} \tag{32}$$

and $$w_2 = \sqrt{w_d^2 - \frac{w_d^2}{4\ Q_d^2}} = w_d\sqrt{1 - \frac{1}{4\ Q_d^2}} \tag{33}$$

Now Q_o may be expected to be greater than 10,000. The term

$$\frac{1}{4\ Q_o^2} = \frac{1}{4 \times (10^4)^2} = 2.5 \times 10^{-9}$$ is very much smaller than 1.

Thus we find that to an accuracy of better than one part in 10^8 w_1 may be defined as

$$w_1 = w_o = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C}\right)} \tag{34}$$

It has been estimated that the minimum value of n will be 250. Referring to Equation 11 we find that the minimum Q with damping is

$$Q_d \doteq 2n \doteq 500$$

Then $$\frac{1}{4\ Q_d^2} = \frac{1}{4\ (500)^2} = 10^{-6} \tag{35}$$

Since this value is very much smaller than 1, Equation 33 becomes, with an error of half a part in a million

$$w_2 = w_d = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C_z}\right)} \tag{36}$$

Thus it is seen that the purely resistive effects of damping have negligible effects on the frequency (less than 1 part per million). It remains to determine the frequency departure which results from the reactance change.

Substituting Equation 26 in Equation 36

$$w_2 = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{2C}\right)} \tag{37}$$

But $C = nC_o$ Therefore

$$w_1 = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{250\ C_o}\right)} = \sqrt{\frac{1}{L_oC_o}\left(1 + \frac{1}{250}\right)} \tag{38}$$

$$w_2 = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{500\ C_o}\right)} = \sqrt{\frac{1}{L_oC_o}\left(1 + \frac{1}{500}\right)} \tag{39}$$

$$\frac{w_2}{w_1} = \frac{\sqrt{1.002}}{\sqrt{1.004}} = .9990 \tag{40}$$

Thus it is seen that, when $n = 250$, the frequency of oscillation is decreased by 1 part per thousand by the addition of the damping resistor. It is pointed out that this study was with reference to the natural frequency of a crystal in conjunction with its holder capacitance and the external added capacitance.

In the other extreme, where $n = 2000$, the frequency of oscillation is decreased by only one-eighth of a part per thousand.

Comparison of damping frequency with operating frequency.

Since it has been possible to determine the frequency change with reference to the crystal frequency, it is necessary to interpret this information with reference to the actual oscillator. A rather detailed analytic and experimental study of Pierce and Miller oscillators has been made by Terry,[1] in which he expresses the ratio of the actual frequency, β , of the oscillator to the crystal frequency, βa. The calculated and experimental curves obtained by Terry are presented in Figure 24 on page 39. Referring to Figure 24, it is seen that $\beta / \beta a > 1$ for a Pierce circuit and that $\beta / \beta a < 1$ for a Miller circuit.

(1) E. M. Terry, "The Dependence of the Frequency of Quartz Piezo-Electric Oscillators Upon Circuit Constants, Proceedings of the Institute of Radio Engineers, Vol. 16, Nov., 1928, p. 1486.

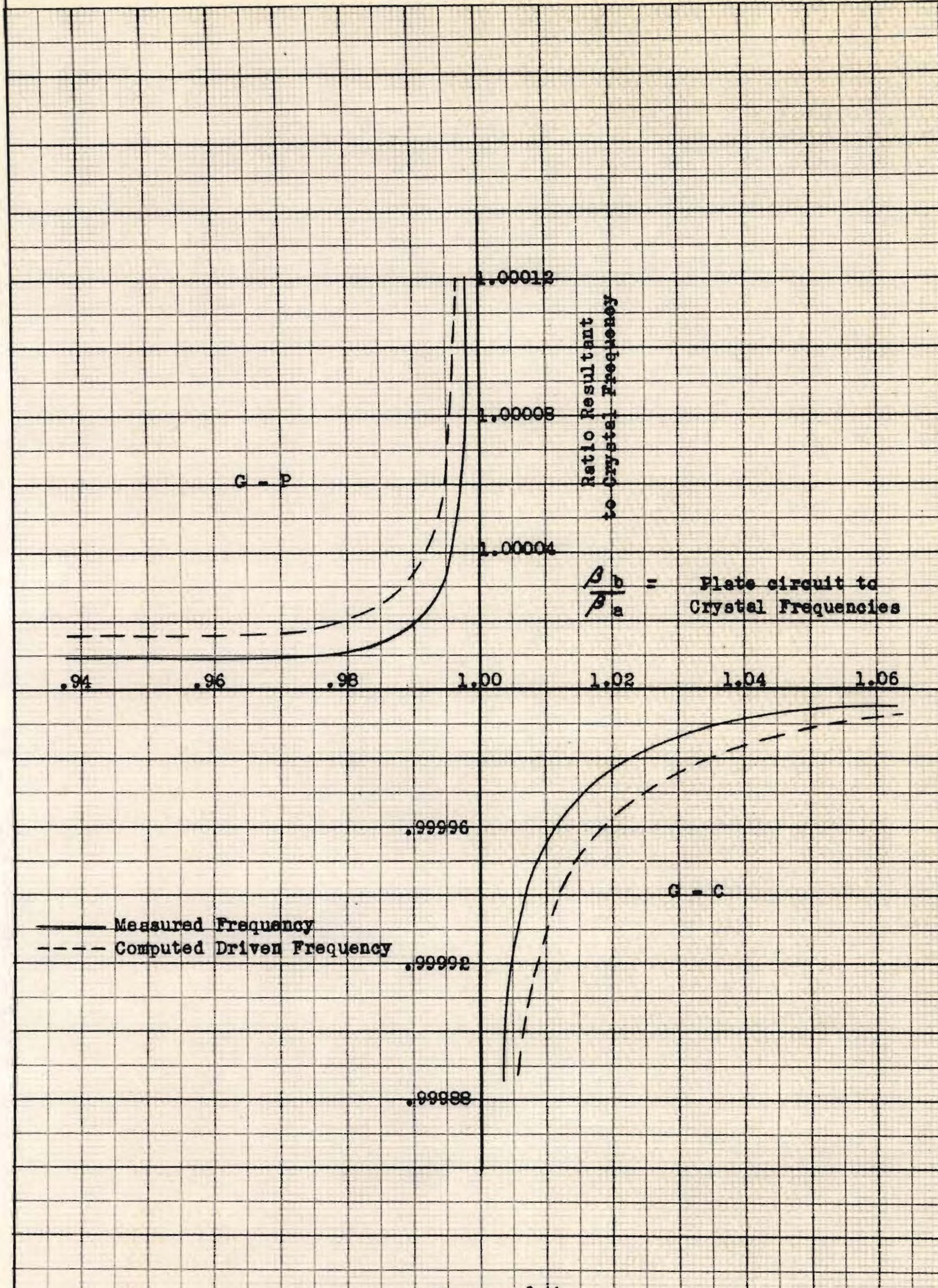

1.00012
1.00008
1.00004
.99996
.99992
.99988
.94
.96
.98
1.00
1.02
1.04
1.06
Ratio Resultant
to Crystal Frequency
G - P
G - C
β_b / β_a = Plate circuit to Crystal Frequencies
Measured Frequency
Computed Driven Frequency

Figure 24

For both the Pierce and Miller oscillators shown in Figure 25, Terry represents the crystal frequency by

$$\beta a = \sqrt{\frac{1}{L_1 Ca}} \tag{41}$$

where $$\frac{1}{Ca} = \frac{1}{C_o} + \frac{1}{C_h} - \frac{C_x}{C_h^2} \tag{42}$$

$$\frac{1}{C_x} = \frac{1}{C_h} + \frac{1}{C_2} + \frac{1}{C} \tag{43}$$

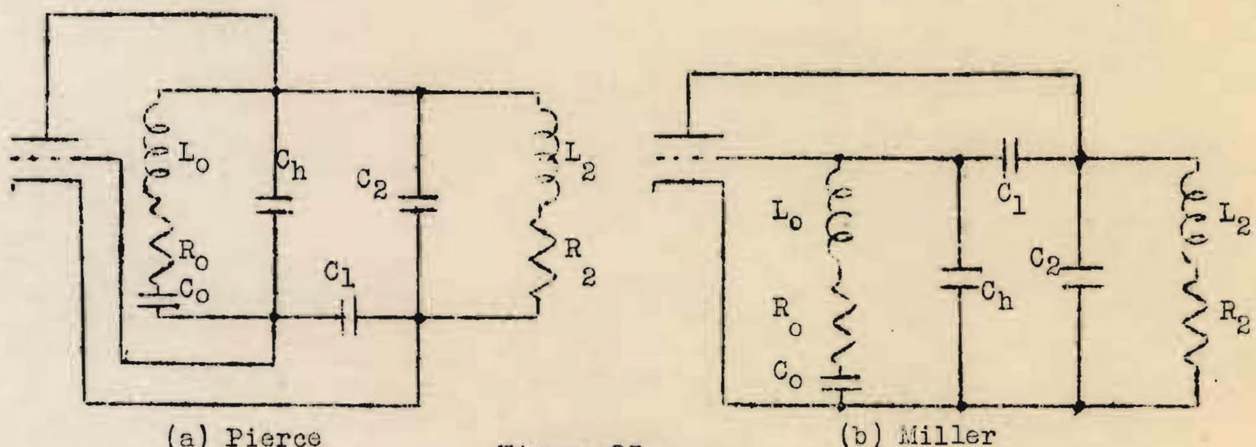


Figure 25.
Simplified Oscillator Circuits.

and β_b is the frequency to which the plate circuit is tuned.

Substituting Equations 42 and 43 in Equation 41 there results

$$\beta a = \sqrt{\frac{1}{L_o}\left[\frac{1}{C_o} + \frac{1}{C_h + \dfrac{C_1C_2}{C_1 + C_2}}\right]} \tag{44}$$

Employing the definition

$$C = C_h + \frac{C_1C_2}{C_1 + C_2} \tag{45}$$

we have

$$\beta_a = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C}\right)} \tag{46}$$

Referring to Equation 34

$$w_1 = \sqrt{\frac{1}{L_o}\left(\frac{1}{C_o} + \frac{1}{C}\right)} = \beta_a \tag{47}$$

There may be some question about neglecting L_2; however, it is pointed out that in the Pierce circuit L_2 is of such a value that $wL_2 \gg \frac{1}{wC_2}$ and L_2 may be neglected without introducing appreciable error. A discussion of the Miller circuit will be omitted temporarily.

By the use of Figure 24 and Equations 34 and 37 it is possible to determine qualitatively the change in frequency when damping is used; this change equals the sum of the frequency change given by Equation 40 and Figure 29. In the preceding paragraph it was stated that $wL_2 \gg \frac{1}{wC_2}$ so the ratio β_b/β_a is considerably less than unity. It is evident that the largest part of the frequency change is indicated by Equation 40. Therefore, the Pierce and Miller circuits are about equal with respect to this type of damping.

IV. FREQUENCY COMPENSATED DAMPING

In Section III it was shown that resistive damping caused a change in frequency. For certain applications this effect will be objectionable, so it is necessary to devise a method to obtain damping without producing a change of frequency. Since the frequency change is caused by an effective change in capacity when the damping resistor is connected, an obvious solution would be to associate inductance with the damping resistance and thereby cancel the capacity change. It has been found that the circuit shown in Figure 26 will accomplish this result when circuit constants are selected

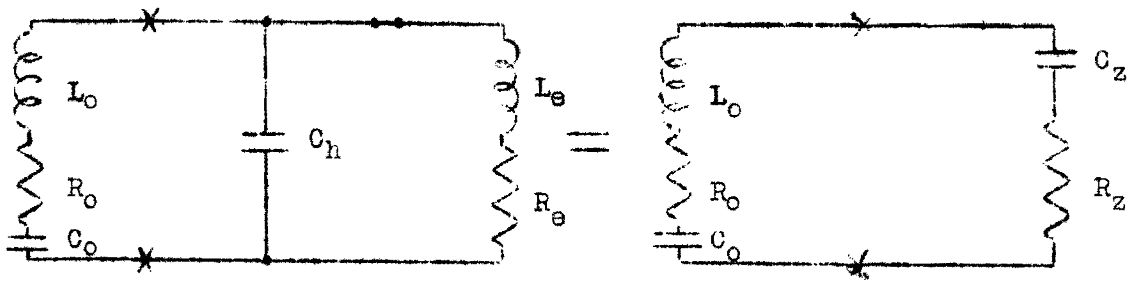


Figure 26.
Equivalent Circuit for Frequency Compensation.

so that

$$L_e = \frac{1}{w^2C} \text{ and } R_e = \frac{1}{wC} \tag{48}$$

Then

$$Z = R_z - j\frac{1}{wC_z} = \frac{\frac{R_e}{w^2C^2} - j\left[\frac{R_e^{\,2}}{wC} + \frac{L_e}{C}\left(wL_e - \frac{1}{wC}\right)\right]}{R_e^{\,2} + \left(wL_e - \frac{1}{wC}\right)^2} \tag{49}$$

Substituting Equation 48

$$R_z - j\frac{1}{wC_z} = \frac{\frac{1}{w^3C^3} - j\left[\frac{1}{w^3C^3} + \frac{L_e}{C}\left(\frac{1}{wC} - \frac{1}{wC}\right)\right]}{\frac{1}{w^2C^2} + \left(\frac{1}{wC} - \frac{1}{wC}\right)^2} \tag{50}$$

$$R_z - j\frac{1}{wC_z} = \frac{1}{wC} - j\frac{1}{wC} \tag{51}$$

Then

$$R_z = \frac{1}{wC} \qquad\qquad C_z = C \tag{52}$$

Thus, since $C_z = C$, there will be no change in frequency when damping is applied. Furthermore, R_z is twice the R_z obtained with resistive damping (see Equation 10).

This change of damping impedance from R to series R_e and L_e necessitates a re-examination of the effect on R_z and C_z of a frequency variation when R_e and L_e are fixed (as determined by one specific frequency).

Let w_o be some frequency for which R_e and L_e are calculated. Then define $w = d\ w_o$. (53)

From Equation 48

$$R_e = \frac{1}{w_o C} \qquad\qquad L_e = \frac{1}{{w_o}^2 C} \tag{54}$$

Substituting Equation 53 and 54 in 50

$$R_z - j\frac{1}{wC_z} = \frac{\frac{1}{w_o^3 C^3 d^2} - j\left[\frac{1}{dw_o^3 C^3} + \frac{d}{w_o^3 C^3} - \frac{1}{dw_o^3 C^3}\right]}{\frac{1}{w_o^2 C^3} + \left(d - \frac{1}{d}\right)^2} \tag{55}$$

$$R_z - j\frac{1}{wC_z} = \frac{1}{w_o C} \cdot \frac{\frac{1}{d^2} - jd}{d^2 - 1 + \frac{1}{d^2}} \tag{56}$$

$$R_z = \frac{1}{w_o C} \cdot \frac{1}{d^4 - d^2 + 1} \qquad (57)$$

$$\frac{C_z}{C} = 1 - \frac{1}{d^2} + \frac{1}{d^4} \qquad (58)$$

The maximum R_z for any frequency is given by

$$R_z = \frac{1}{wC} = \frac{1}{dw_o C} \qquad (59)$$

and the desired ratio $\frac{C_z}{C}$ is 1.

Equations 57 and 59 are plotted in Figure 27 with d as the abscissa and $w_o C R_z$ as the ordinate. Equation 58 is plotted in Figure 28 with d as the abscissa and C_z/C as the ordinate.

It can be seen that this method of damping is critical with respect to frequency. It is believed that a further examination of this method will reveal a way to make frequency compensated damping less dependent on the operating frequency.

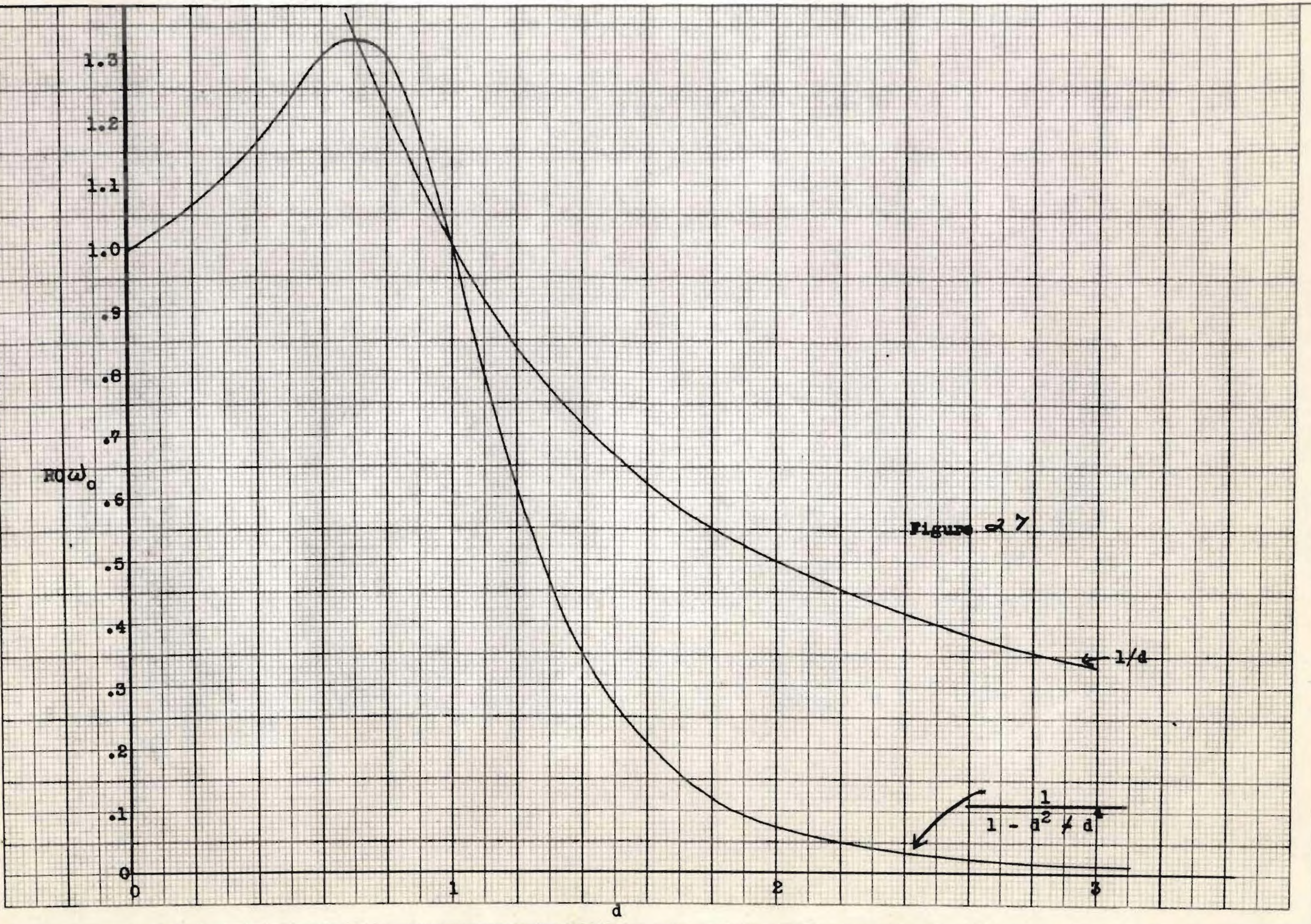


Figure 27

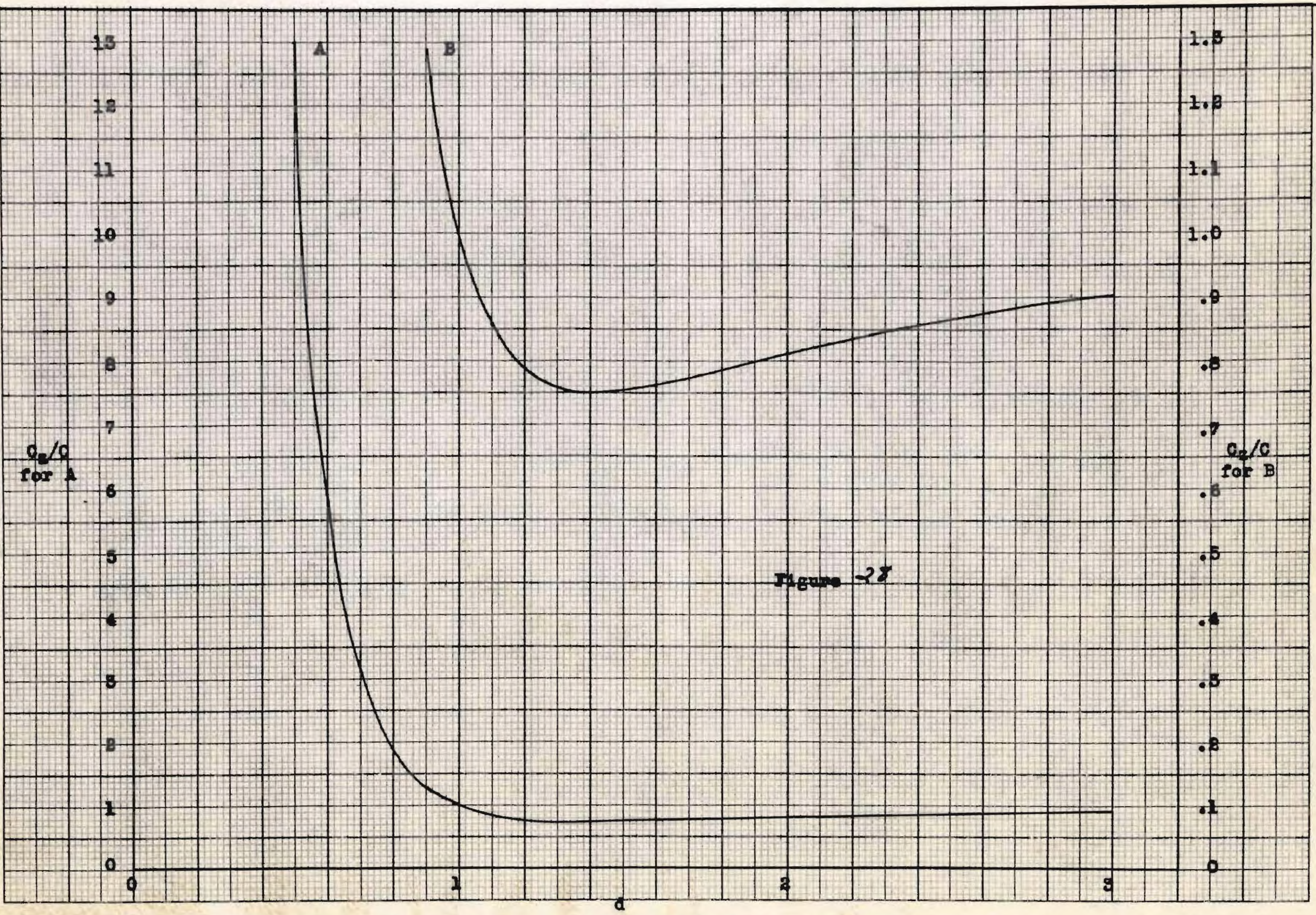

A
B
13
12
11
10
9
8
7
6
5
4
3
2
1
0
C_s/C
for A
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
C_z/C
for B
0
1
2
3
d

Figure 28

V. PLATE CIRCUIT DAMPING IN PIERCE OSCILLATORS

The initial attempts to damp directly a crystal in the Pierce circuit were failures. A circuit diagram of this method is shown in Figure 29. At the time these experiments were undertaken, relays which would key at 50 dot-cycles per second were not available, so various electronic keying methods were tried. It was found when a vacuum tube was used as a switch, the necessity for providing D.C. biases to the tube required the use of circuits so complicated as to be impractical.

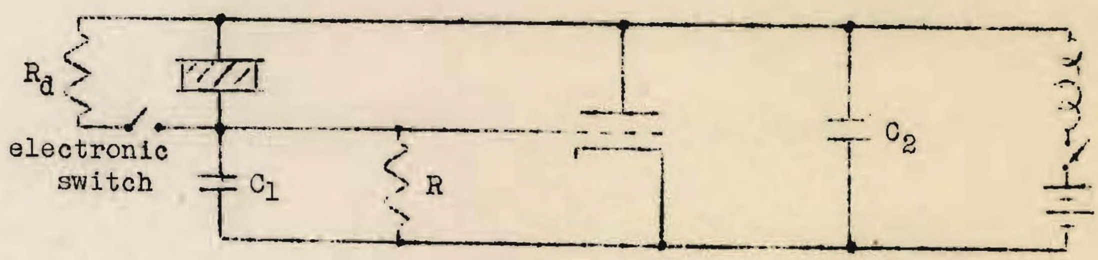


Figure 29.
Direct Damping of Crystal.

A careful reconsideration of the problem indicated that a considerable degree of damping of the crystal could be accomplished by placing the damping resistor in parallel with C_2. The basic circuit is shown in Figure 30.

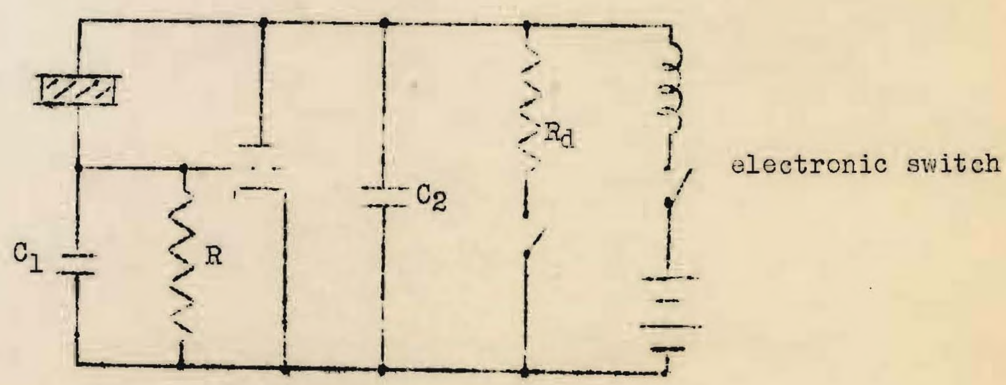


Figure 30.
Plate Circuit Damping.

Several preliminary experiments were performed to indicate the feasibility of this method.

The actual circuit is shown in Figure 31, and the keying waveforms obtained for both the damped and undamped conditions are shown in Figure 32. It is seen that satisfactory keying waveforms can be obtained for keying rates up to 60 dot-cycles per second. On the basis of these preliminary results, it was decided to conduct an analytical examination of this method. However, before this examination was completed, high-speed relays became available, and, in addition, a Pierce circuit was developed which had one of the crystal terminals grounded. Because of these reasons and the fact that a continued study became increasingly difficult, further examination was suspended.

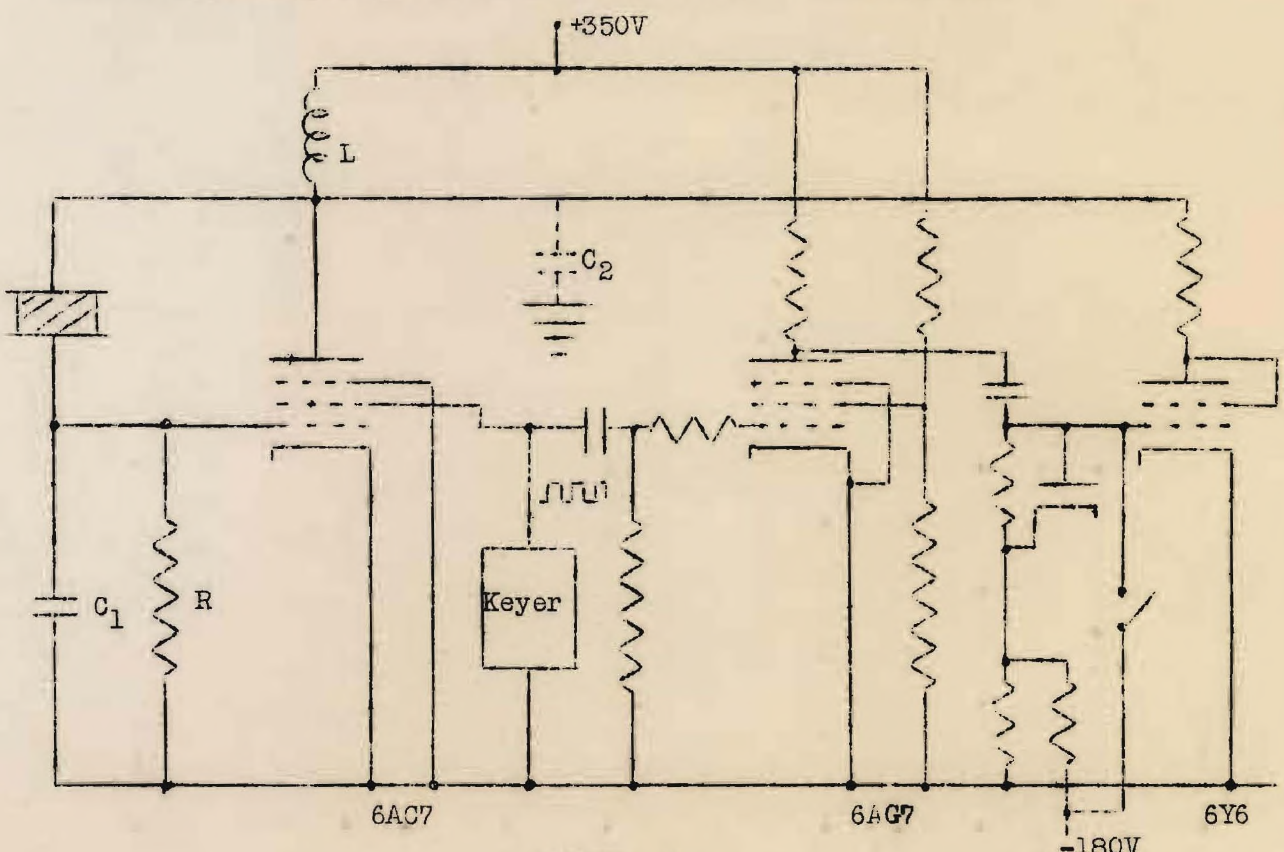
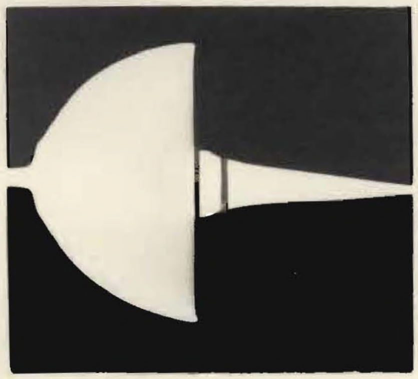


Figure 31.
Circuit for Plate Circuit Damping.

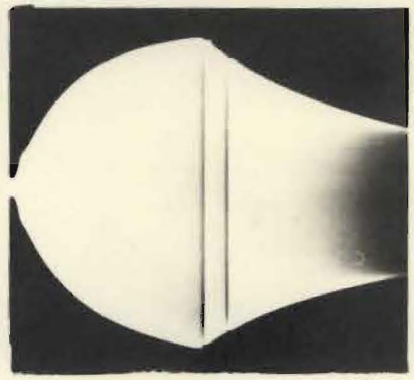
Damped

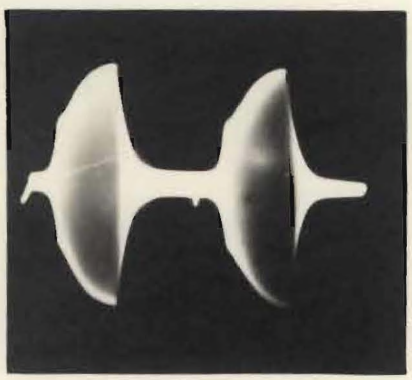
Undamped

Crystal #84, 1910 kc.
Approximately 1.5 milli-
seconds between dark
lines.

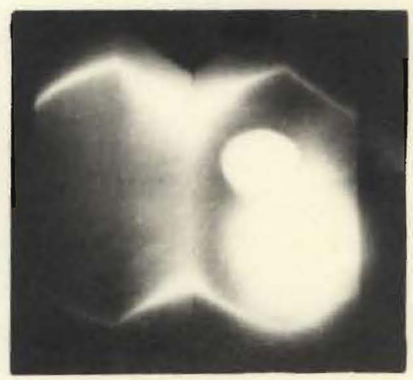


Crystal #84, 1910 kc.
Above pictures magnified.
Approximately 1.5 milli-
seconds between dark
lines.





Crystal #92, 2410 kc.
Keyed at 60 dot-cycles
per second.



Crystal #13, 3680 kc.
Keyed at 60 dot-cycles
per second.

Figure 32
Plate Circuit Damping

Analysis of plate circuit damping.

There follows the partial examination of the damping method shown in Figure 30. Since L (Figure 30) is large, the circuit may be reduced to that in Figure 33a and subsequently to that in 33b. The damping produced by the grid leak is negligible. Accordingly, the grid leak is not shown in Figure 33. For convenience the subscript is dropped from the damping resistor.

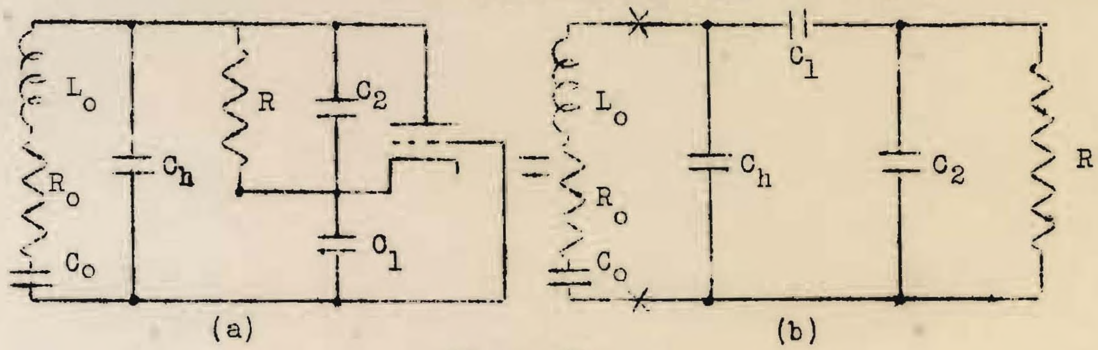


Figure 33.
Equivalent Circuit for Plate Damping.

Breaking at XX, there results

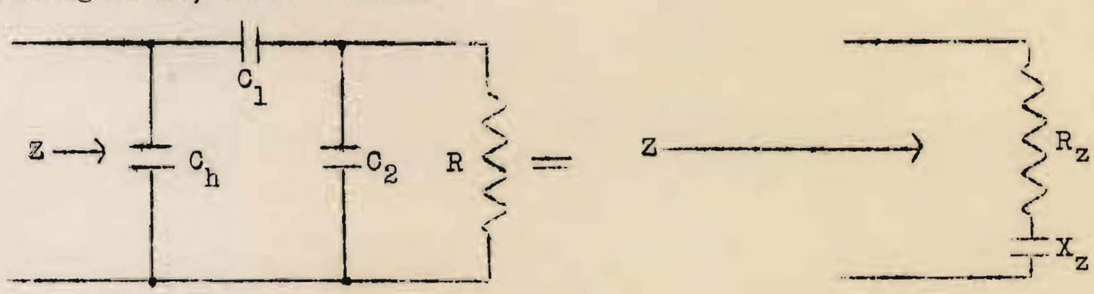


Figure 34.
Circuit Transformations.

Now impose the following conditions

$$C_2 = \frac{C_1}{b} \quad \text{and} \quad C_1 = (b + 1)\,K \tag{60}$$

where $K = \dfrac{C_1 C_2}{C_1 + C_2}$ is the specified load capacitance.

Let $X = \frac{1}{wC_1} = \frac{1}{wK\ (b + 1)}$ $\quad X_h = \frac{1}{wC_h}$ $\quad bX = \frac{1}{wC_2}$ (61)

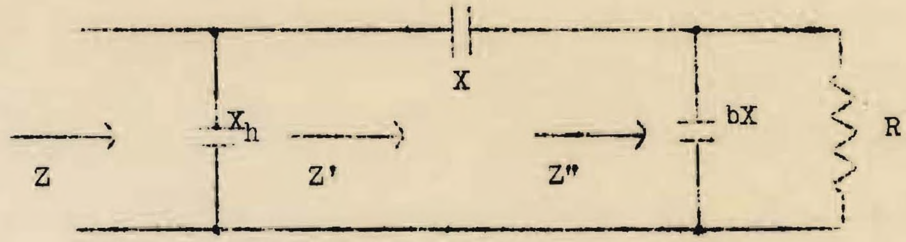


Figure 35.
Network to be Reduced.

The network of Figure 35 may be solved for R_z

$$Z" = \frac{-jbXR}{R - jbX} = \frac{b^2X^2R - jbXR^2}{R^2 + b^2X^2} \tag{62}$$

$$Z' = Z" - jX = \frac{b^2X^2R - j\left[(b + 1)XR^2 + b^2X^3\right]}{R^2 + b^2X^2} \tag{63}$$

$$Z = \frac{-jX_h\left(b^2X^2R - j\left[(b + 1)\ XR^2 + b^2X^3\right]\right)}{-j\left[(b + 1)\ XR^2 + b^2X^3\right] - jX_h\left[R^2 + b^2X^2\right] + b^2X^2R} \tag{64}$$

$$Z = \frac{-\left[(b + 1)\ XX_hR^2 + b^2X^3X_h\right] - jb^2X^2X_hR}{b^2X^2R - j\left[(b + 1)\ XR^2 + X_hR^2 + b^2X^3 + b^2X^2X_h\right]} \tag{65}$$

Rationalize and solve for R_z

$$R_z = \frac{b^2X^2X_h^2\left[R^3 + b^2X^2R\right]}{b^4X^4R^2 + \left(R^2\left[(b + 1)\ X + X_h\right] + b^2X^2(X + X_h)\right)^2} \tag{66}$$

It is seen that Equation 66 involves three independent variables, namely, b, X, and X_h. Since a load capacitance, K (32 μμ fds.), has been specified as standard, it is feasible to define X_h in the following manner

$$X_h = \frac{1}{wC_h} = \frac{1}{waK} \quad \text{where } a = \frac{C_h}{K} \tag{67}$$

Substituting Equations 61 and 67 in Equation 66 and simplifying

$$R_z = \frac{\frac{b^2}{w^2K^2\ (b+1)^2(a+1)^2}\left[R^3 + \frac{b^2}{w^2K^2\ (b+1)^2} R\right]}{R^4 + \frac{b^2}{w^2K^2(b+1)^2(a+1)^2}\left[\frac{2(a+1)(a+b+1)}{b+1} + \frac{a^2b^2}{(b+1)^2}\right]R^2 + \frac{b^4}{w^4K^4(b+1)^4(a+1)^2}\left[\frac{a+b+1}{b+1}\right]^2} \quad (68)$$

Determination of maximum available damping rates.

To obtain maximum damping R_z must be a maximum. The classical method of obtaining a maximum R_z is to set $\frac{dR_z}{dR} = 0$. However, when Equation 68 is differentiated, the resulting equation is a cubic with literal coefficients. The solution of a complicated literal cubic equation is a laborious and usually unsatisfactory process.

Since the conditions $C_2 = \frac{C_1}{b}$, $C_1 = (b+1)K$, and $C_h = bK$ were imposed, there are, besides R, three variables, a, b, and w.

Make these definitions

$$B = \frac{b^2}{(b+1)^2} \qquad C = \frac{1}{(a+1)^2}$$

$$D = \left[\frac{2(a+1)(a+b+1)}{b+1} + \frac{a^2b^2}{(b+1)^2}\right] \quad (69)$$

$$E = \left[\frac{a+b+1}{b+1}\right]$$

Substituting Equation 69 in Equation 68

$$R_z = \frac{\frac{1}{w^2K^2}\ BC\left[R^3 + \frac{B}{w^2K^2} R\right]}{R^4 + \frac{BCD}{w^2K^2} R^2 + \frac{B^2\ C\ E^2}{w^4K^4}} \quad (70)$$

$$\frac{dR}{dR} = \frac{\frac{1}{w^2K^2}\ BC\left(\left[3R^2 + \frac{B}{w^2K^2}\right]\left[R^4 + \frac{BCD}{w^2K^2} + \frac{B^2\ C\ E^2}{w^4K^4}\right] - \left[R^3 + \frac{B}{w^2K^2} R\right]\left[4R^3 + \frac{2\ BCD}{w^2K^2} R\right]\right)}{\left[R^4 + \frac{BCD}{w^2K^2} R^2 + \frac{B^2\ C\ E^2}{w^4K^4}\right]^2} \quad (71)$$

The value of R for maximum R_z is given by the equation

$$R^6 + \frac{1}{w^2K^2}\left[3B - BCD\right] R^4 + \frac{1}{w^4K^4}\left[B^2CD - 3B^2CE^2\right] R^2 - \frac{1}{w^6K^6} B^3CE^2 = 0 \qquad (72)$$

This equation when factored will assume the form

$$\left[R^2 - G^2\frac{1}{w^2K^2}\right]\left[R^4 + MR^2\frac{1}{w^2K^2} + N\frac{1}{w^4K^4}\right] = 0 \qquad (73)$$

or

$$R = G \cdot \frac{1}{wK} \qquad (74)$$

where G is a yet undetermined coefficient.

The other roots are complex conjugate and are spurious roots introduced in the rationalization processes.

An expression has been obtained for R; next obtain an expression for R_z. Substitute Equation 74 in Equation 70

$$R_z = \frac{\frac{BC}{w^2K^2}\left[\frac{G^3}{w^3K^3} + \frac{BG}{w^3K^3}\right]}{\frac{G^4}{w^4K^4} + \frac{BCDG^2}{w^4K^4} + \frac{B^2CE^2}{w^4K^4}} \qquad (75)$$

$$R_z = \frac{1}{wK}\left[\frac{BC\,(G^3 + BG)}{G^4 + BCDG^2 + B^2E^2C}\right] = \frac{1}{wK} \cdot H \qquad (76)$$

Now G and H are functions of a and b. Various values of a and b were used to calculate G and H, the results being tabulated in Tables 6 and 7 and plotted in Figures 36 and 37, respectively.

Now

$$Q_d = \frac{wL_o}{R_z} = \frac{wL_o}{\frac{H}{wK}} = \frac{w^2KL_o}{H} \qquad (77)$$

R for			
b	a = .25	a = .5	a = .75
1	.4494 $\frac{1}{wk}$	.4171 $\frac{1}{wK}$	.3924 $\frac{1}{wK}$
2	.5779 $\frac{1}{wK}$	.5187 $\frac{1}{wK}$	.4764 $\frac{1}{wK}$
3	.6372 $\frac{1}{wK}$	.5621 $\frac{1}{wK}$	.5089 $\frac{1}{wK}$
5	.6943 $\frac{1}{wK}$	.6017 $\frac{1}{wK}$	.5357 $\frac{1}{wK}$
8	.7309 $\frac{1}{wK}$	.6253 $\frac{1}{wK}$	.5505 $\frac{1}{wK}$
12	.7530 $\frac{1}{wK}$	.6387 $\frac{1}{wK}$	.5577 $\frac{1}{wK}$
∞	.800 $\frac{1}{wK}$	.6667 $\frac{1}{wK}$	.571 $\frac{1}{wK}$

Table 6.
Values of R for Various Values of a and b.

R_Z for			
b	a = .25	a = .5	a = .75
1	.1776 $\frac{1}{wK}$	.1333 $\frac{1}{wK}$	.1039 $\frac{1}{wK}$
2	.246 $\frac{1}{wK}$	.1900 $\frac{1}{wK}$	.1522 $\frac{1}{wK}$
3	.282 $\frac{1}{wK}$	.222 $\frac{1}{wK}$	.1803 $\frac{1}{wK}$
5	.320 $\frac{1}{wK}$	.256 $\frac{1}{wK}$	.211 $\frac{1}{wK}$
8	.346 $\frac{1}{wK}$	.281 $\frac{1}{wK}$	.234 $\frac{1}{wK}$
12	.362 $\frac{1}{wK}$	.296 $\frac{1}{wK}$	.249 $\frac{1}{wK}$
∞	.400 $\frac{1}{wK}$	.333 $\frac{1}{wK}$	.285 $\frac{1}{wK}$

Table 7.
Values of R_Z for Various Values of a and b.

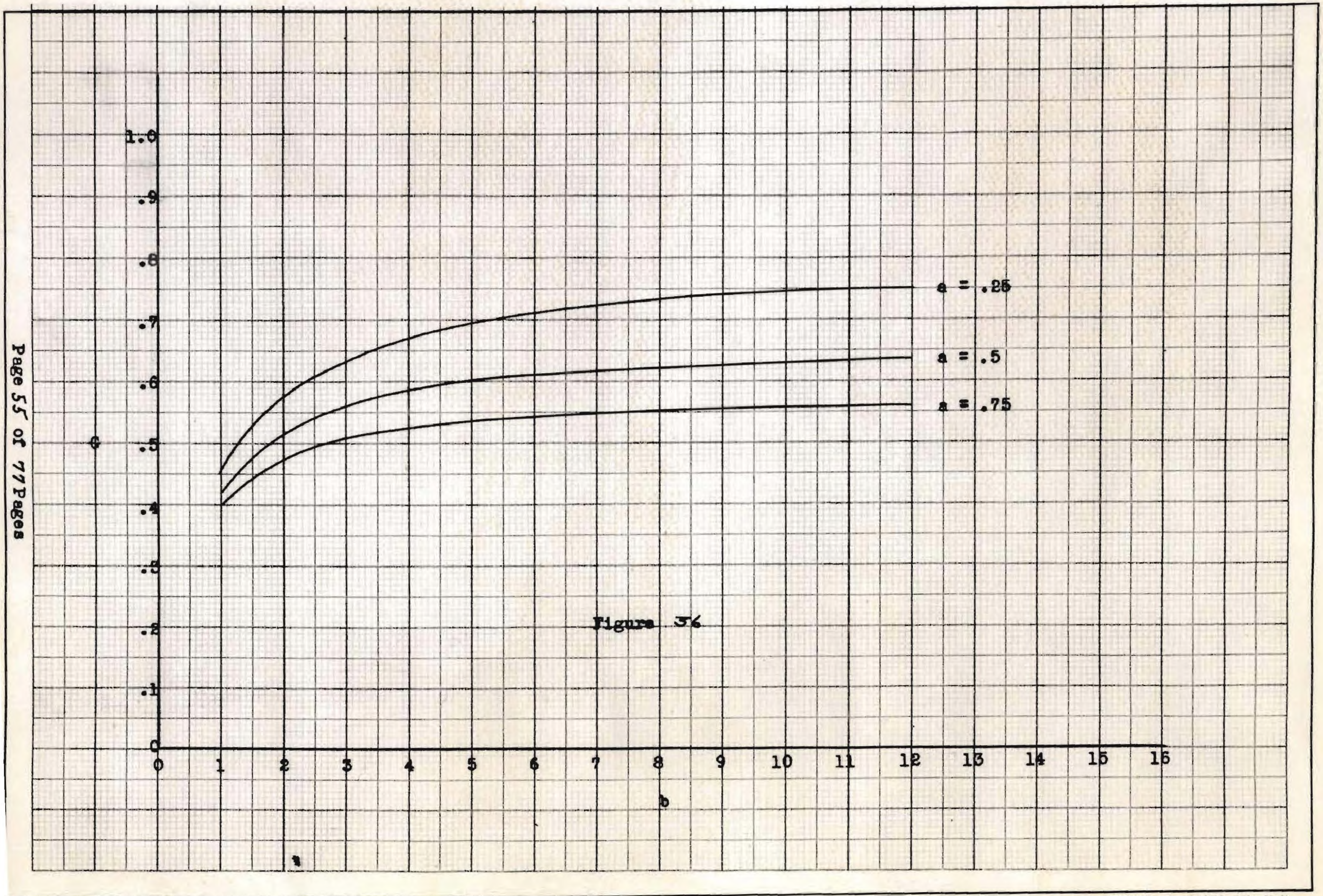


Figure 36

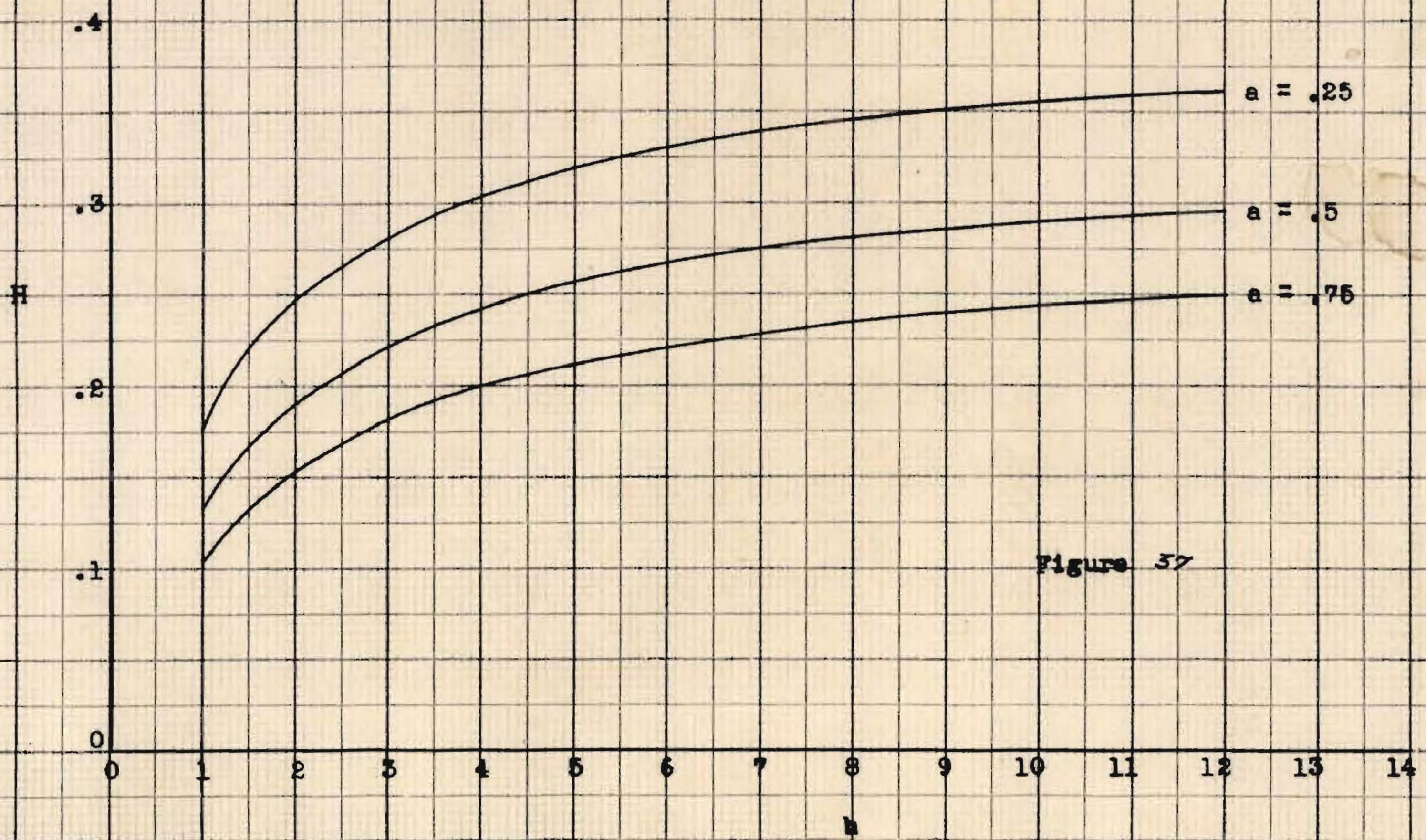


Figure 37

$$\text{and } \triangle = \frac{4.34w}{Q_d} \text{ db/sec} = \frac{4.34H}{wKL_o} \text{ db/sec} \tag{78}$$

From Equation 78 it is seen that the higher frequencies are unfavorable to rapid damping.

At this point it becomes necessary to clarify and interpret the information obtained.

It has been found that the rise time is not noticeably changed by varying b from 1/2 to 2 and that for b = 5 the rise time is increased by only 120%. When using high gain tubes, it has been found that the fall time is the limiting factor of high speed keying. For this reason it is feasible to sacrifice rapid rise time to obtain a shorter fall time. An optimum value of b would seem to be 5, because no serious limitation has been placed on the rise time, and the fall time cannot be decreased to any great extent by making b > 5.

Thus it is seen that this method of damping will work fairly well. A continuation of this examination would require a determination of the effects of damping on the frequency of oscillations, the effect of a change of crystal frequency on damping for a fixed value of R, and the effect of variation of holder capacity on damping. For the reasons previously mentioned this study was discontinued at this point. It is believed that this method might be applicable to the modification of existing equipment, if the transmitter circuits are such that a frequency change during damping is unimportant.

VI. CRYSTAL DAMPING - DIFFERENTIAL EQUATION SOLUTION

The damping of oscillations in a crystal may be increased by connecting a resistance across the crystal terminals. Moreover, there is strong evidence that the addition of an inductance in series with the damping resistor results in a considerable increase in the damping and reduces the deviations of frequency from the original resonance frequency of the crystal.

Previous approaches through complex algebra did not give adequate information on the amount of damping which can be so obtained.or the accompanying frequency departures. Accordingly, a solution through differential equations was attempted.

Our analysis will be confined to a study of the behavior of a crystal during the time that the damping circuit is connected to its terminals. It should be noted that C_2 includes not only the shunting capacitance of the crystal but also any external capacitance added by the working circuit. Referring to Figure 38, it can be shown that i_1 represents the mechanical motion of the crystal, so that the decay of i_1 is a measure of crystal damping. We proceed, therefore, to solve for i_1.

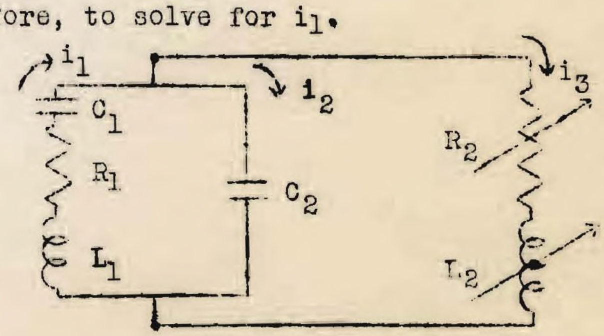


Figure 38.
Equivalent Damping Circuit.

Kirchhoff equations.

$$i_3 = i_1 - i_2 \tag{79}$$

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \int \frac{i_1}{C_1}\,dt + \int \frac{i_2}{C_2}\,dt = 0 \tag{80}$$

$$L_2 \frac{di_3}{dt} + R_2 i_3 - \int \frac{i_2}{C_2}\,dt = 0 \tag{81}$$

Elimination of i_2 and i_3.

From Equation 80

$$\frac{-i_2}{C_2} = L_1 \frac{d^2 i_1}{dt^2} + R_1 \frac{di_1}{dt} + \frac{i_1}{C_1} \tag{82}$$

From Equations 79 and 81

$$L_2 \frac{di_1}{dt} + R_2 i_1 = \int \frac{i_2}{C_2}\,dt + L_2 \frac{di_2}{dt} + R_2 i_2 \tag{83}$$

Differentiating Equation 83

$$L_2 \frac{d^2 i_1}{dt^2} + R_2 \frac{di_1}{dt} = \frac{i_2}{C_2} + L_2 \frac{d^2 i_2}{dt^2} + R_2 \frac{di_2}{dt} \tag{84}$$

From Equations 82 and 84

$$(L_1 + L_2) \frac{d^2 i_1}{dt^2} + (R_1 + R_2) \frac{di_1}{dt} + \frac{i_1}{C_1} = L_2 \frac{d^2 i_2}{dt^2} + R_2 \frac{di_2}{dt} \tag{85}$$

From Equation 82

$$-i_2 = C_2 \left(L_i \frac{d^2 i}{dt^2} + R_i \frac{di_1}{dt} + \frac{i_1}{C_1}\right) \tag{86}$$

From Equations 85 and 86

$$(L_1 + L_2) \frac{d^2 i_1}{dt^2} + (R_1 + R_2) \frac{di_1}{dt} + \frac{i_1}{C_1} + L_2 C_2 \left[L_1 \frac{d^4 i_1}{dt^4} + R_1 \frac{d^3 i_1}{dt^3} + \frac{1}{C_1} \frac{d^2 i_1}{dt^2}\right]$$
$$+ R_2 C_2 \left[L_1 \frac{d^3 i_1}{dt^3} + R_1 \frac{d^2 i_1}{dt^2} + \frac{1}{C_1} \frac{di_1}{dt}\right] = 0 \tag{87}$$

Equation 87 is a differential equation in i_1 **alone.** We now use i instead of i_1 and group the terms

$$L_1L_2C_2 \frac{d^4i}{dt^4} + (R_1L_2C_2 + R_2L_1C_2)\frac{d^3i}{dt^3} + \left[\frac{L_2C_2}{C_1} + R_1R_2C_2 + L_1 + L_2\right]\frac{d^2i}{dt^2}$$

$$+\left[\frac{R_2C_2}{C_1} + R_1 + R_2\right]\frac{di}{dt} + \frac{i}{C_1} = 0 \qquad (88)$$

If L_2 is not zero (and our analysis is on the basis of finite L_2), we may divide out by the coefficient of $\frac{d^4i}{dt^4}$, obtaining the more useful form

$$\frac{d^4i}{dt^4} + \left[\frac{R_1}{L_1} + \frac{R_2}{L_2}\right]\frac{d^3i}{dt^3} + \left[\frac{1}{L_1C_1} + \frac{R_1}{L_1}\cdot\frac{R_2}{L_2} + \frac{1}{L_2C_2} + \frac{1}{L_1C_2}\right]\frac{d^2i}{dt^2}$$

$$+\left[\frac{1}{L_1C_1}\cdot\frac{R_2}{L_2} + \frac{1}{L_2C_2}\cdot\frac{R_1}{L_1} + \frac{1}{L_2C_2}\cdot\frac{R_2}{L_1}\right]\frac{di}{dt} + \frac{i}{L_1C_1\cdot L_2C_2} = 0 \qquad (89)$$

The quantities L_1, R_1, C_1, and C_2 are parameters of the crystal. They are fixed for a given crystal. The quantities R_2 and L_2 are independent variables. The frequency or frequencies of oscillation which will result from the closing of the crystal terminals **through** R_2 and L_2 in series, and the associated damping terms of these frequencies, are the dependent variables. We define the following quantities.

<u>Basic definitions.</u>

Let

$$\left.\begin{aligned} n &= \frac{C_2}{C_1} \\ \alpha_1 &= \frac{R_1}{2L_1} \qquad \alpha_2 = \frac{R_2}{2L_2} \\ w_1^2 &= \frac{1}{L_1C_1} \qquad w_2^2 = \frac{1}{L_2C_2} \end{aligned}\right\} \qquad (90a)$$

$$\left.\begin{aligned} H &= \frac{\alpha_1}{w_1} \qquad k = \frac{\alpha_2}{w_1} \\ Q_1 &= \frac{w_1}{2\alpha_1} \qquad Q_2 = \frac{w_1}{2\alpha_2} \\ &= \text{nominal } Q \text{ of the coil at frequency } w_1 \end{aligned}\right\} \tag{90b}$$

In addition let $R_2 = \frac{rR_1}{n}$ and $L_2 = \frac{SL_1}{n}$

whence $r = \frac{R_2 n}{R_1}$ $\qquad S = \frac{L_2 n}{L_1}$ $\qquad$ (91)

We then have the relationships

$$\left.\begin{aligned} w_2^2 &= \frac{w_1^2}{S} \\ \alpha_2 &= \frac{r\alpha_1}{S} \qquad \text{Also} \quad Q_2 = \frac{1}{2k} \\ \frac{r}{S} &= \frac{k}{H} \qquad\qquad \frac{Q_2}{Q_1} = \frac{H}{k} \end{aligned}\right\} \tag{92}$$

Direct substitution of appropriate quantities into Equation 89 yields

$$\frac{d^4i}{dt^4} + 2(\alpha_1 + \alpha_2)\frac{d^3i}{dt^3} + (w_1^2 + w_2^2 + \frac{w_1^2}{n} + 4\alpha_1\alpha_2)\frac{d^2i}{dt^2}$$

$$+ 2(w_1^2\alpha_2 + w_2^2\alpha_1 + w_2^2\alpha_1 \cdot \frac{r}{n})\frac{di}{dt} + (w_1^2 w_2^2)i = 0 \tag{93}$$

Further manipulation gives

$$\frac{d^4i}{dt^4} + 2w_1(H + k)\frac{d^3i}{dt^3} + w_i^2(1 + \frac{1}{S} + \frac{1}{n} + 4Hk)\frac{d^2i}{dt^2}$$

$$+ 2w_1^3(k + \frac{H}{S} + \frac{k}{n})\frac{di}{dt} + \frac{w_1^4}{S} i = 0 \tag{94}$$

On account of the presence of two LC pairs in the circuit, we expect a solution for i_1 of the form

$$i_1 = I_1' e^{-\alpha t} \sin(\beta t + \phi_1') + I_1'' e^{-\gamma t} \sin(\delta t + \phi_1'') \tag{95}$$

Then the auxiliary or p equation associated with the linear differential Equation 94 would be

$$(p + \alpha + j\beta)(p + \alpha - j\beta)(p + \gamma + j\delta)(p + \gamma - j\delta) = 0 \tag{96}$$

Writing the differential Equation 94 as

$$\frac{d^4 i}{dt^4} + D\frac{d^3 i}{dt^3} + E\frac{d^2 i}{dt^2} + F\frac{di}{dt} + G = 0 \tag{97}$$

and equating coefficients with the expanded p equation, we get

$$2(\alpha + \gamma) = D \tag{98}$$

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 4\alpha\gamma = E \tag{99}$$

$$2(\alpha\gamma^2 + \alpha\delta^2 + \gamma\alpha^2 + \gamma\beta^2) = F \tag{100}$$

$$\alpha^2\gamma^2 + \alpha^2\delta^2 + \beta^2\gamma^2 + \beta^2\delta^2 = G \tag{101}$$

Now the differential equation is solved when the fourth degree algebraic equation in p is solved. We have found that the latter is not easily accomplished in the form of Equations 98-101 owing to widely different magnitudes of the real and imaginary components of its roots, which refer to the damping terms and frequencies, respectively, of the solution for i_1 (Equation 95). This difference in the order of magnitude is, of course, due to the high intrinsic Q of the crystal. Accordingly, we enlarge the real components of the roots of p by the following transformation

Let

$$\left.\begin{array}{ll} \alpha = akw_1 & \gamma = gkw_1 \\ \beta = bw_1 & \delta = fw_1 \end{array}\right\} \qquad (102)$$

We now solve for the new variables

$$\begin{array}{ll} a = \dfrac{\alpha}{\alpha_2} & g = \dfrac{\gamma}{\alpha_2} \\ b = \dfrac{\beta}{w_1} & f = \dfrac{\delta}{w_1} \end{array} \qquad (103)$$

The transformations have been so chosen that these new variables are all of the order of magnitude of unity.

Equations 98-101 become, as a consequence

$$2kw_1 \,(a + g) = D \qquad (104)$$

$$w_1{}^2 \left[k^2(a^2 + 4ag + g^2) + b^2 + f^2\right] = E \qquad (105)$$

$$2kw_1{}^3 \left[k^2ag(a + g) + af^2 + gb^2\right] = F \qquad (106)$$

$$w_1{}^4 \,(g^2k^2 + f^2)(a^2k^2 + b^2) = G \qquad (107)$$

From the differential Equation 94 we have that

$$D = 2w_1 \,(H + k) = 2kw_1 \left[\frac{H}{k} + 1\right] \qquad (108)$$

$$E = w_1{}^2 \left(1 + \frac{1}{S} + \frac{1}{n} + 4Hk\right) \qquad (109)$$

$$F = 2w_1{}^3 \left[k + \frac{H}{S} + \frac{k}{n}\right] = 2kw_1{}^3 \left[1 + \frac{1}{r} + \frac{1}{n}\right] \qquad (110)$$

$$G = \frac{w_1{}^4}{S} \qquad (111)$$

Equating the separate Equations 104-107 and 108-111 gives

$$a + g = \frac{H}{k} + 1 \tag{112}$$

$$k^2(a^2 + 4ag + g^2) + b^2 + f^2 = 1 + \frac{1}{S} + \frac{1}{n} + 4Hk \tag{113}$$

$$k^2 ag\,(a + g) + af^2 + gb^2 = 1 + \frac{1}{r} + \frac{1}{n} \tag{114}$$

$$(g^2k^2 + f^2)(a^2k^2 + b^2) = \frac{1}{S} \tag{115}$$

Equations 112-115 are exact, and their simultaneous solution gives a, b, g, and f as functions of H and n (crystal constants) and k, r, and S (terms involving the two independent variables R_2 and L_2).

Because of the orders of magnitudes of certain quantities which apparently hold universally for crystals, we may make some simplifying assumptions. The simple fact that all the parameters are positive quantities is helpful in this connection.

Thus, the ratio $n = \frac{C_2}{C_1}$ is very large for crystals, being 1500 in the case of our representative "Standard Crystal"[1], and can never be less than 125 because of the inherent properties of crystalline quartz. We therefore neglect $\frac{1}{n}$ as compared to 1.

The ratio $\frac{H}{k} = \frac{Q_2}{Q_1}$ is very small, since the Q of the crystal Q_1 is always very large (41,700 for "Standard Crystal"), while a physically realizable coil has $Q_2 = 100$ or thereabouts. Since the deviation of frequencies β and δ from the crystal frequency w_1 is expected to be slight, we may assume that $\frac{Q_2}{Q_1}$

---

(1) For convenience in comparing various results, a "Standard Crystal" having the following properties has been used:

$f = 3.2$ mc	$C_1 = 0.04$ mmf	$w_1 = 20 \times 10^6$
$R_1 = 30$ ohms	$C_2 = C_h + K = 28 + 32$ mmf	
$L_1 = 1/16$ henries	$Q_1 = 41,700$	

is of the order $\frac{1}{400}$ for the Standard Crystal.

On the same basis, $k \ll \frac{1}{4H}$ or $4H\,k \ll 1$.

Hence we neglect $\frac{H}{k}$ and $4H\,k$ in comparison to 1.

Employing these approximations, Equations 112-115 become

$$a + g = 1 \tag{116}$$

$$k^2(a^2 + 4ag + g^2) + b^2 + f^2 = 1 + \frac{1}{S} \tag{117}$$

$$k^2 ag(a + g) + af^2 + gb^2 = 1 + \frac{1}{r} \tag{118}$$

$$(g^2k^2 + f^2)(a^2k^2 + b^2) = \frac{1}{S} \tag{119}$$

We note that in the Equations 116-119 both r and S appear exclusively in reciprocal form.

For convenience therefore, let

$$\left.\begin{aligned} t &= \frac{1}{r} \\ u &= \frac{1}{S} \end{aligned}\right\} \tag{120}$$

Now, inasmuch as a and g (the damping term variables) should not differ too widely and we would like to be able to control their ratio more readily, we make one more purely arbitrary transformation to further the solution of Equations 116-119.

Let

$$\left.\begin{aligned} g &= \ell a, \\ f &= mb \end{aligned}\right\} \tag{121}$$

Equations 116-119 become

$$a(\ell + 1) = 1 \tag{122}$$

$$k^2a^2(\ell^2 + 4\ell + 1) + b^2(m^2 + 1) = u + 1 \tag{123}$$

$$a\left[k^2a^2\ell(\ell + 1) + b^2(m^2 + \ell)\right] = t + 1 \tag{124}$$

$$(\ell^2k^2a^2 + m^2b^2)(k^2a^2 + b^2) = u \tag{125}$$

We proceed to solve this system for a, b, ℓ, and m with the independent variables k, t, and u. Note that $t = \frac{uH}{k}$. (126)

For convenience the unknowns of the original differential equation are listed here in terms of the various quantities introduced.

$$\left.\begin{array}{ll} \alpha = akw_1 & \gamma = \ell akw_1 = \ell\alpha \\ \beta = bw_1 & \delta = mbw_1 = m\beta \end{array}\right\} \tag{127}$$

<u>Solution of the system given in Equations 122-125.</u>

From Equation 122

$$a = \frac{1}{\ell + 1} \tag{128}$$

From Equation 123

$$b^2 = \frac{(u + 1)(\ell + 1)^2 - k^2(\ell^2 + 4\ell + 1)}{(m^2 + 1)(\ell + 1)^2} \tag{129}$$

Substituting Equations 128 and 129 in Equation 124

$$\frac{k^2\ell(\ell + 1)}{(\ell + 1)^2} + \frac{\left[(u + 1)(\ell + 1)^2 - k^2(\ell^2 + 4\ell + 1)\right]}{(m^2 + 1)(\ell + 1)^2}(m^2 + \ell) = (t + 1)(\ell + 1)$$

$$k^2\ell(\ell + 1)(m^2 + 1) + (u + 1)(\ell + 1)^2(m^2 + \ell) - k^2(\ell^2 + 4\ell + 1)(m^2 + \ell) - (t + 1)(\ell + 1)^3(m^2 + 1) = 0 \tag{130}$$

Solving for m

$$m^2\left[k^2\ell(\ell+1) + (u+1)(\ell+1)^2 - k^2(\ell^2+4\ell+1) - (t+1)(\ell+1)^3\right]$$
$$+\left[k^2\ell(\ell+1) + \ell(u+1)(\ell+1)^2 - k^2\ell(\ell^2+4\ell+1) - (t+1)(\ell+1)^3\right] = 0 \qquad (131)$$

$$m^2 = -\frac{\left\{k^2\ell^2(\ell+3) + (\ell+1)^2\left[\ell(t-u) + (t+1)\right]\right\}}{\left\{k^2(3\ell+1) + (\ell+1)^2\left[\ell(t+1) + (t-u)\right]\right\}} \qquad (132)$$

By the same approximations made to obtain Equations 116-119, since $\frac{u}{t} = \frac{k}{H}$ and $H \ll k$, it follows that $t \ll u$. From the way we have carried on the solution we may choose ℓ either larger or smaller than unity. The choice will merely reverse which pair of roots are identified with α and β. Let us for convenience choose $\ell \geq 1$, whereupon we have $t \ll \ell u$ because $t \ll u$. We neglect t compared with u and ℓu in the expression for m^2, obtaining

$$m^2 = \frac{k^2\ell^2(\ell+3) - (\ell+1)^2(\ell u - 1)}{k^2(3\ell+1) + (\ell+1)^2(u - \ell - \ell t)} \qquad (133)$$

Substituting Equations 128 and 129 in Equation 125

$$\frac{k^4\ell^2}{(\ell+1)^4} + \frac{k^2(m^2+\ell^2)\left[(u+1)(\ell+1)^2 - k^2(\ell^2+4\ell+1)\right]}{(m^2+1)(\ell+1)^4}$$
$$+\frac{m^2\left[(u+1)(\ell+1)^2 - k^2(\ell^2+4\ell+1)\right]^2}{(m^2+1)^2(\ell+1)^4} = u \qquad (134)$$

$$k^4\ell^2(m^2+1)^2 + k^2(m^2+1)(m^2+\ell^2)(u+1)(\ell+1)^2 - k^4(m^2+1)(m^2+\ell^2)$$
$$(\ell^2+4\ell+1) + m^2(u+1)^2(\ell+1)^4 - 2m^2k^2(u+1)(\ell+1)^2(\ell^2+4\ell+1)$$
$$+ m^2k^4(\ell^2+4\ell+1)^2 - u(m^2+1)^2(\ell+1)^4 = 0 \qquad (135)$$

Solving for m

$$m^4\left[k^4\ell^2 + k^2(u+1)(\ell+1)^2 - k^4(\ell^2+4\ell+1) - u(\ell+1)^4\right]$$
$$+ m^2\Big[2k^4\ell^2 + k^2(u+1)(\ell+1)^2(\ell^2+1) - k^4(\ell^2+1)(\ell^2+4\ell+1)$$
$$+ (u+1)^2(\ell+1)^4 - 2k^2(u+1)(\ell+1)^2(\ell^2+4\ell+1) + k^4(\ell^2+4\ell+1)^2$$
$$- 2u(\ell+1)^4\Big] + \Big[k^4\ell^2 + k^2\ell^2(u+1)(\ell+1)^2 - k^4\ell^2(\ell^2+4\ell+1)$$
$$-u(\ell+1)^4 = 0 \qquad (136)$$

$$m^4\left[k^4(4\ell+1) - k^2(u+1)(\ell+1)^2 + u(\ell+1)^4\right] - m^2\Big[2k^4\ell(2\ell^2+9\ell+2)$$
$$- k^2(u+1)(\ell+1)^2(\ell^2+8\ell+1) + (u^2+1)(\ell+1)^4\Big]$$
$$+ \Big[k^4\ell^3(\ell+4) - k^2\ell^2(u+1)(\ell+1)^2 + u(\ell+1)^4 = 0 \qquad (137)$$

Tabulating these Equations we have

$$a = \frac{1}{\ell+1} \qquad (128) -- (138)$$

$$b^2 = \frac{(u+1)(\ell+1)^2 - k^2(\ell^2+4\ell+1)}{(m^2+1)(\ell+1)^2} \qquad (129) -- (139)$$

$$m^2 = \frac{k^2\ell^2(\ell+3) - (\ell+1)^2(\ell u - 1)}{k^2(3\ell+1) + (\ell+1)^2(u-\ell-\ell t)} \qquad (133) -- (140)$$

$$m^4\left[k^4(4\ell+1) - k^2(u+1)(\ell+1)^2 + u(\ell+1)^4\right]$$
$$-m^2\left[2k^4\ell(2\ell^2+9\ell+2) - k^2(u+1)(\ell+1)^2(\ell^2+8\ell+1) + (u^2+1)(\ell+1)^4\right]$$
$$+ \Big[k^4\ell^3(\ell+4) - k^2\ell^2(u+1)(\ell+1)^2 + u(\ell+1)^4 = 0 \qquad (137) -- (141)$$

Equations 138, 139, 140, and 141 may be most readily solved by assuming a value for ℓ, then a value for <u>k</u>; and from the resulting numerical equations 140 and 141 finding all real positive values of <u>u</u> and corresponding values of <u>m^2</u>. <u>a</u> and <u>b</u> are then determined by Equations 138 and 139, and Equation 127 gives the solutions for α, β, γ, and δ.

Equations 122, 123, 124, and 125 offer a direct check on the results.

In the future the foregoing method will be used to plot α, β, γ, and δ versus $S = \frac{1}{u}$ for various values of $Q_2 = \frac{1}{2k}$, as an aid in design engineering. At the moment the only existing solution is an exploratory numerical example.

Example.

Choosing $\ell = 2$, and $k = 0.1$, we have by combining Equations 140 and 141, $u = 1.15$ which is the only real positive root. Because u has already been chosen as a positive real number we know that this is the desired solution. Then from Equation 140

$$m^2 \doteq 1.42 \qquad m = 1.19 \tag{142}$$

From Equations 139

$$b^2 \doteq 0.885 \qquad b = 0.94 \tag{143}$$

From Equation 138

$$a = \frac{1}{3} \tag{144}$$

In order to check the correctness of these numbers we substitute directly in Equations 122-125.

$$\frac{1}{3}(3) = 1 \quad \text{----- should be } 1 \tag{145}$$

$$.014 + .885\ (2.42) \doteq 2.14 \quad \text{----}\ \ 2.15 \tag{146}$$

$$\frac{1}{3}\left[.01 + .885\ (3.42)\right] \doteq 1.01 \quad \text{----}\ 1.00 \tag{147}$$

$$(.004 + 1.26)(.001 + .885) \doteq 1.12 \quad \text{----}\ 1.15 \tag{148}$$

The above approximate equalities indicate the essential correctness of the previous solution.

From Equation 127

$$\left.\begin{array}{ll} \alpha = \dfrac{w_1}{30} & \gamma = \dfrac{w_1}{15} \\ \beta = 0.94\, w_1 & \delta = 1.12\, w_1 \end{array}\right\} \qquad (149)$$

The circuit Q's corresponding to these two frequencies are

$$\left.\begin{array}{l} Q' = \dfrac{\beta}{2\alpha} = 14.1 \\ Q'' = \dfrac{\delta}{2\gamma} = 8.4 \end{array}\right\} \qquad (150)$$

(Compare these with the undamped Q of the "Standard Crystal" $Q_1 = 41,700$.)

The above results were due to insertion of a coil having inductance

$L_2 = \dfrac{L_1}{un} = 0.87\,\dfrac{L_1}{n}$ and nominal Q (at w_1) of 5.

From an examination of numerical solutions we may say that:

1. The higher the Q of the coil, the less the deviation from w_1 in the **in the** frequencies β and δ ;
2. As the Q of the coil is decreased, the value of the coil inductance L_2 must be decreased to maintain a given ratio of damping terms α and γ associated with the new frequencies β and δ ;
3. The two frequencies β and δ lie on opposite sides of w_1 in magnitude.

VII. DISCUSSION OF OSCILLATOR CIRCUITS

In the preceding sections it has been shown that crystal damping must be employed if high keying rates are used. In the Miller circuit one terminal of the crystal is grounded, thus simplifying the application of a damping network. In the Pierce circuit generally used, both terminals of the crystal are at relatively high R.F. potentials. It is considered desirable to use a modified version of the Pierce circuit, two examples of which are shown in Figure 39.

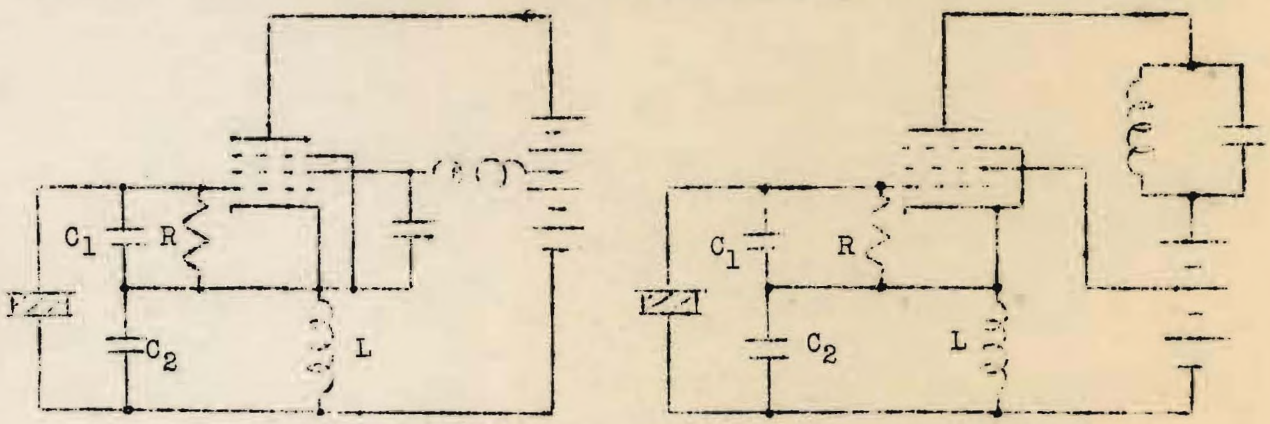


Figure 39.
Modified Pierce Oscillator Circuits.

In this way it is possible to connect one of the crystal terminals to ground. With the use of available high speed relays it is believed that satisfactory damping can be obtained.

It has been found that the attainment of short rise times requires the use of high gain tubes. In addition, the circuits used should effectively use all the g_m of the tube to build up oscillations. In the conventional E.C.O. Miller and Pierce circuits, only the control to screen grid g_m is effective in building up and maintaining oscillations. Since the screen g_m is much smaller than the plate g_m, it is believed that conventional E.C.O. circuits will have too long a rise time to perform properly at high keying rates. On the other hand, it can be shown that the oscillator in Figure 39 uses both the plate and screen g_m's to build up oscillations. It should be possible through the proper use of this circuit to obtain all the advantages of electron coupling as well as that of having one terminal of the crystal grounded.

Series resonant operation.

Another circuit which is believed to have possibilities for short rise times is shown in Figure 40.

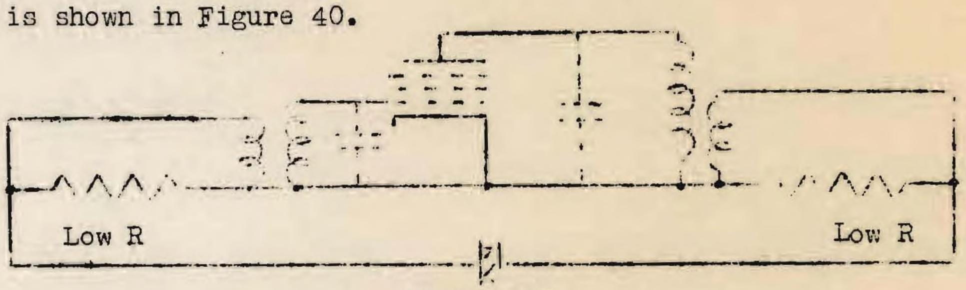


Figure 40.
Transformer Coupled Series Resonant Oscillator.

A crystal operating at its series resonant frequency has a low impedance; however, vacuum tubes are high impedance generators. Thus, it is seen that there is an inherent mismatch of impedances when using a series resonant crystal directly with a vacuum tube. The circuit in Figure 40 uses transformers to match the vacuum tube to the crystal and permit both to operate at optimum impedance conditions. Furthermore, by making the output impedance of T_2 and the input impedance of T_1 considerably lower than the lowest impedance of the crystal, the crystal should control the frequency very effectively.

A preliminary calculation indicates that relatively short rise and fall times can be secured in this circuit by use of conventional tubes and reasonable transformation ratios. Moreover, the circuit should operate over a considerable band of frequencies by substituting crystals without tuning the elements.

VIII. OBJECTIVES FOR FINAL QUARTER

The final quarter will be devoted to completion of the project as outlined in the contract. Specific topics yet to be worked on include:

1. Experimental study of resistive and frequency compensated resistive damping.
2. Theoretical and experimental study of impedance matched series resonant oscillator.
3. Determination of best tubes to use in Miller and modified Pierce circuits.
4. Reduction of theoretical and experimental results to engineering design formulas and recommendations.
5. Experimental study of extreme damping rates.
6. Final report.

Respectfully submitted,

William A. Edson
Project Director

Approved:

Gerald A. Rosselot
Director

APPENDIX A - PRECISION TIMER

The observation of large numbers of rise and fall times requires some simple and precise method of measurement.

The method employed is similar to one developed by Lange[1] to study high Q cavities. Essentially it consists of a precision delay multivibrator and associated circuits which give an output of two negative pulses. The first pulse is used as a zero time marker; the second pulse is delayed from the first by a known time. This output is used to blank the trace of an oscilloscope.

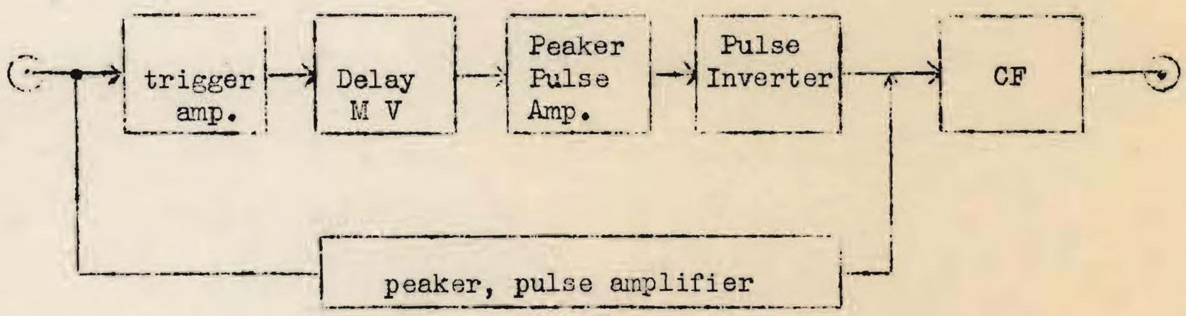


Figure 1.
Block Diagram of Timer.

Figure 1 is a block diagram of the timer, the complete circuit diagram being shown in Figure 5. The output of the keyer is applied to the trigger amplifier which is biased so that the leading edge of the keying pulse triggers the delay multivibrator. After a measured time the multivibrator flops back to the original condition; thus, the output of the delay multivibrator is a positive pulse of known time duration. This pulse is peaked and the resultant pulse peaks amplified, and inverted by an inverter which is biased so that it responds only to the peak representing the trailing edge of the multivibrator

(1) Lange, R. W., "Measurement of High Q Cavities at 10,000 Megacycles," Electrical Engineering. December, 1946.

output pulse. This peak is used for the time delay measurement. The important respect in which this circuit differs from others is the fact that the period depends upon the setting of a high grade variable air condenser.

In addition, the input from the keyer is peaked and amplified by an amplifier tube biased so that it responds only to the leading edge of the keying pulse. This peak is used for a zero time marker.

These two peaks are added and applied to a cathode follower. Thus the output of the cathode follower consists of two negative pulses; the first is the zero time marker, and the second is the measuring marker.

The physical appearance of the timer is shown in Figure 2. All controls

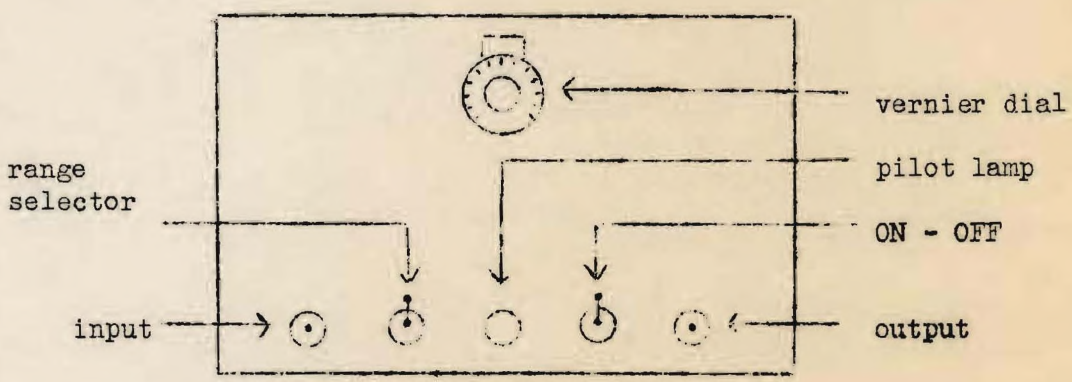


Figure 2.
View of Keyer Panel.

are self-explanatory except the vernier dial and the range selector. The vernier dial is calibrated from 0 to 100 and is used with a calibration chart so that dial readings may be translated to time delays. Turning the vernier dial changes the time interval between the zero time marker and the measuring marker.

The range selector permits measurement on two ranges, namely, .9-13 milliseconds and .1 to 1.2 milliseconds.

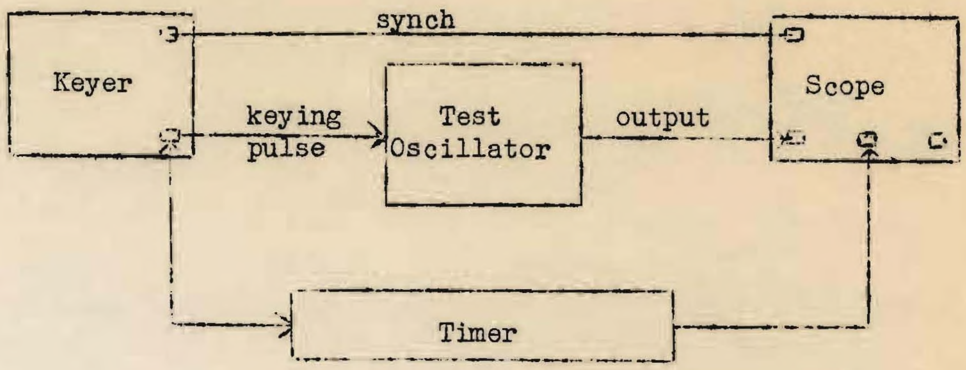


Figure 3.
Typical Experimental Set-Up.

Figure 3 is an experimental set-up in which the timer is used to measure oscillator rise time. The oscillogram obtained will be similar to the one shown in Figure 4.

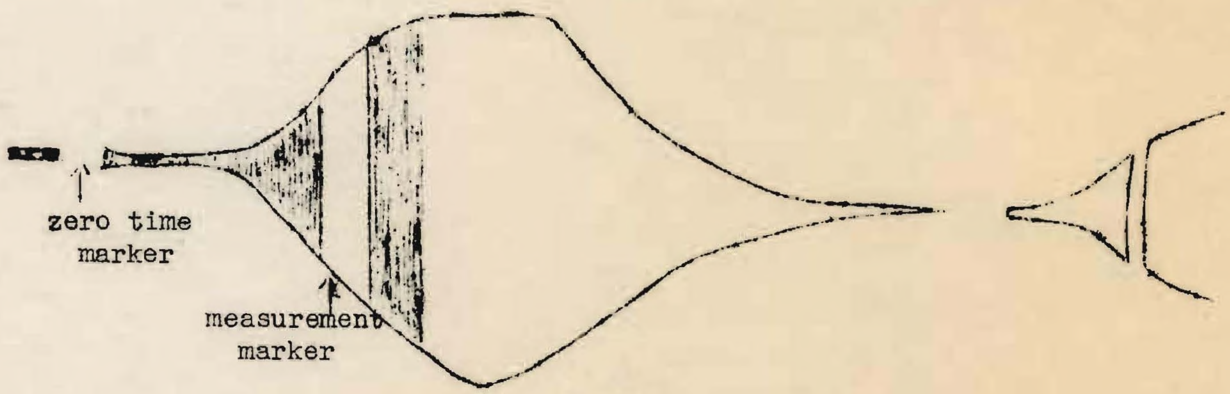
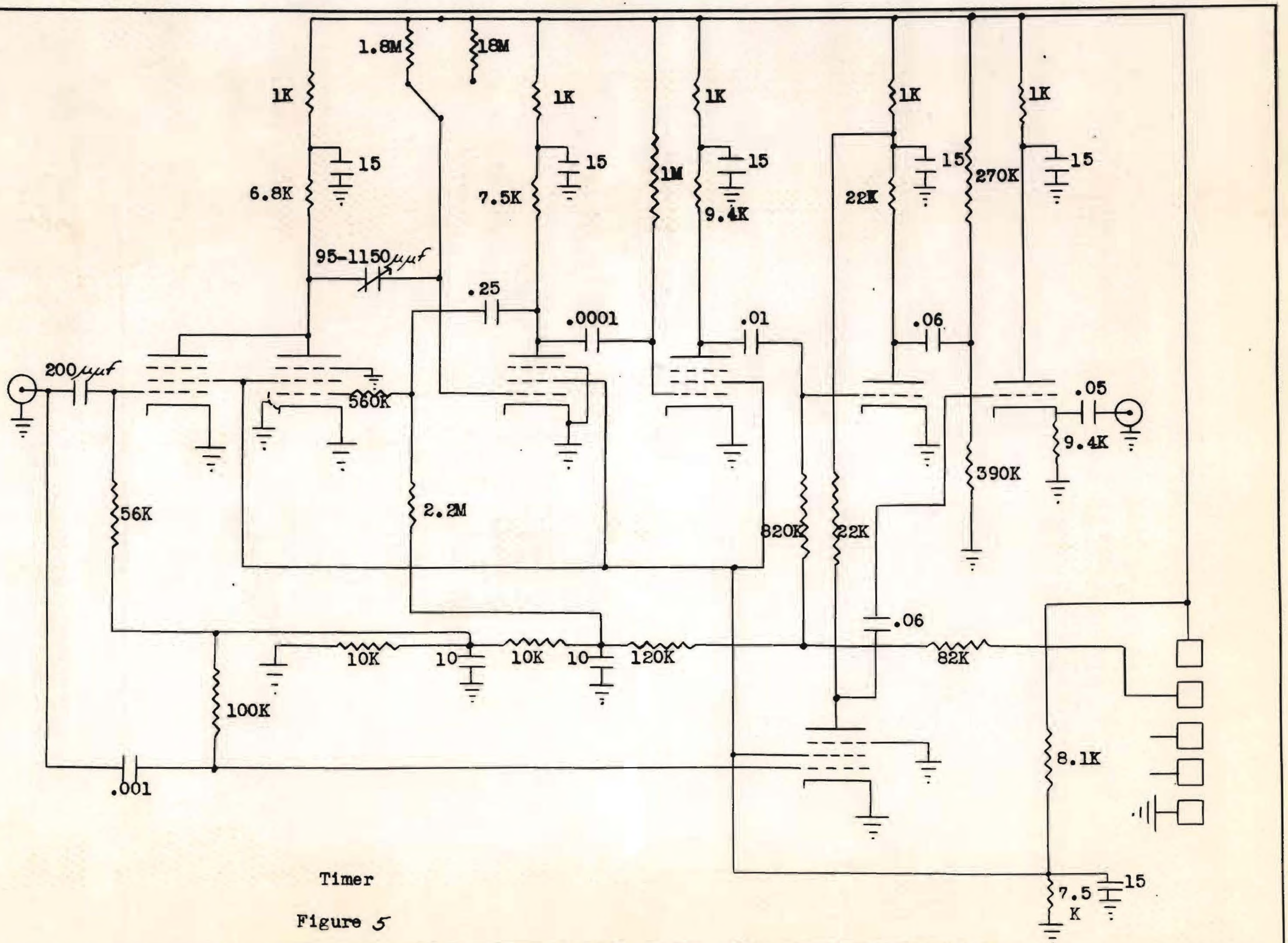


Figure 4.
Oscillogram.

Similarly the timer may be used to measure fall times if the output of the keyer is inverted before being applied to the timer.

In the event another of these timers is constructed, it would be advisable to provide a separate voltage divider for each screen grid and to increase the size of the decoupling filters.



Timer

Figure 5

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PROGRESS REPORT NO. 5

PROJECT NO. 106-6

CONTRACT NO. W36-039-sc-32100

THE KEYING PROPERTIES OF QUARTZ CRYSTAL OSCILLATORS

By

WILLIAM A. EDSON

JULY 31, 1947

GEORGIA SCHOOL OF TECHNOLOGY
THE STATE ENGINEERING EXPERIMENT STATION
ATLANTA, GEORGIA

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PROGRESS DURING FIFTH QUARTER

During this quarter the problem of electrically damping a vibrating crystal plate has been substantially solved. All of the methods developed show that the ratio of the crystal holder capacitance to the equivalent series capacitance is a prime factor limiting the degree of damping which can be obtained. The first method in which the crystal is damped by a resistance connected across the crystal terminals has been presented as a theoretical development in previous reports. Experimental results have been obtained which validate the theoretical development; however, it has not been possible to measure the frequency shift which is indicated by the theory.

A second damping method, which uses an RLC network, produces damping comparable to the resistance method but with no inherent frequency shift. This network provides very wide band operation for fixed values of the network parameters.

The third method uses an LR network and produces extreme damping rates at a fixed frequency. For ordinary capacitance ratios the *Q* of the system is approximately 30. However, to secure the highest damping rates, together with negligible frequency shift, it is necessary to adjust the elements, particularly the inductance, with considerable care.

An extremely difficult problem was encountered in trying to incorporate these damping networks into actual oscillators. Various mechanical (relays) and electronic (vacuum tube switches) methods were tried. It was found that all commercially available relays which have high operating speeds also possess high intercontact capacitances. As a result extremely long rise times were obtained. Electronic methods also proved to be infeasible.

An experiment was performed to determine the tube characteristics which control buildup rates in keyed oscillators. The results of this experiment were not conclusive, although it was repeated in an attempt to eliminate apparent parasitic effects.

In view of the above problems, it was decided to re-examine the damping philosophy. After a study of the problem, it appeared that the best method of attack involved considering the oscillator as an indivisible unit which is keyed by controlling one or more of the D.C. biases. It is believed that this approach will provide a satisfactory solution to the keying problem.

Considerable thought has been devoted to various negative resistance devices and their characteristics. From this there has resulted the development of two transitron oscillators in which the crystal is operated at series resonance. It is hoped that these circuits may be applicable to keyed as well as continuous wave applications.

Extensive tests reveal that the rate and quality of keying which may be achieved in the Pierce, Miller, and Electron-coupled oscillators are substantially independent of the point at which the key is connected. Apparently the keying transient had little effect on the start and buildup of oscillations.

A careful consideration of both old and new oscillator circuits indicates that the behavior is greatly dependent upon the impedance level as well as the capacitance ratio and the Q of the crystal. It is believed that revision and closer control of the impedance level of crystals may be necessary for best performance in oscillators for both keyed and continuous wave operation.

OBJECTIVES FOR FINAL PERIOD

The final period will be devoted to completion of the project as outlined in the contract. Specific topics to be covered include:

1. Further study of keying theory from the viewpoint of the complete oscillator.
2. Detailed study of the two transitron-type series resonant oscillators.
3. Detailed study of the impedance-matched series-resonant oscillator.
4. Reduction of theoretical and experimental results to engineering design formulas and recommendations.
5. Final report.

Respectfully submitted,

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